3D coordinates and vectors in 3D

124 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-3d-coordinates-and-vectors/

Date

A Do Now

Quick questions to start.

A1 From Year 1 [3 marks]

$\mathbf{a} = 3\mathbf{i} - 4\mathbf{j}$. Find $|\mathbf{a}|$ and a unit vector in the direction of $\mathbf{a}$.

Answer .....

A2 Needed today [3 marks]

P is the point $(2, -1)$ and Q is the point $(7, 11)$. Find $\overrightarrow{PQ}$ in terms of $\mathbf{i}$ and $\mathbf{j}$, and the distance PQ.

Answer .....

A3 Needed today [2 marks]

A triangle has sides 5 cm, 7 cm and 8 cm. Find the angle opposite the side of length 7 cm.

Answer .....

A4 Common slip [2 marks]

$\overrightarrow{OA} = 4\mathbf{i} + \mathbf{j}$ and $\overrightarrow{OB} = \mathbf{i} + 5\mathbf{j}$. Find $\overrightarrow{AB}$.

Answer .....

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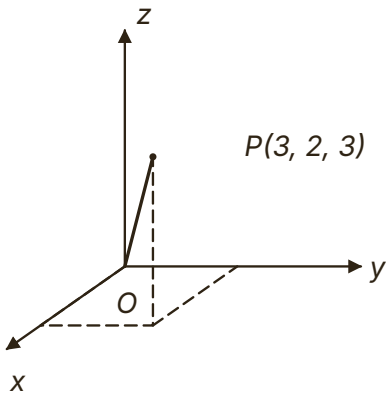
B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find the distance between the points $(1, -2, 4)$ and $(3, 4, 1)$.



Hint The distance from O to (x, y, z) is $\sqrt{x^2 + y^2 + z^2}$, as in the diagram. For two points, use the differences in x, y and z.

B1 Find the distance between the points $A(-1, 4, 2)$ and $B(3, -4, 3)$. [3 marks]

Answer .....

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EXAMPLE 2

The point $P(3, k, 1)$ is 6 units from $Q(5, 1, -3)$. Find the possible values of k.

Hint Write the distance squared in terms of k and set it equal to 36. Expect two answers.

B2 The point $A(k, 2, -1)$ is 6 units from $B(4, -2, 1)$. Find the possible values of k. [4 marks]

Answer .....

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EXAMPLE 3

$\mathbf{a} = 2\mathbf{i} - \mathbf{j} + 2\mathbf{k}$. Find $|\mathbf{a}|$, a unit vector in the direction of $\mathbf{a}$, and the angle $\mathbf{a}$ makes with the positive x-axis.

Hint The angle with the x-axis has $\cos\theta = \dfrac{x\text{-component}}{|\mathbf{a}|}$. Keep the sign of the component.

B3 $\mathbf{b} = 4\mathbf{i} + 4\mathbf{j} - 7\mathbf{k}$. Find $|\mathbf{b}|$, a unit vector in the direction of $\mathbf{b}$, and the angle $\mathbf{b}$ makes with the positive z-axis, to 1 decimal place. [5 marks]

Answer .....

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EXAMPLE 4

A, B and C are the points $(1, 2, 0)$, $(3, 3, 2)$ and $(1, 5, 4)$. Find angle BAC and the area of triangle ABC.

Hint Find the three side lengths from $\overrightarrow{AB}$, $\overrightarrow{AC}$ and $\overrightarrow{BC}$. Then use the cosine rule and area $= \frac{1}{2}bc\sin A$.

B4 [6 marks]

P, Q and R are the points $(2, -1, 3)$, $(4, 1, 4)$ and $(-1, 3, 3)$. Find angle QPR, to 1 decimal place, and the area of triangle PQR, to 3 significant figures.

Answer .....

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EXAMPLE 5

$\mathbf{a} = 2\mathbf{i} - 3\mathbf{j} + \mathbf{k}$ and $\mathbf{b} = 6\mathbf{i} + p\mathbf{j} + q\mathbf{k}$ are parallel. Find p and q.

Hint Parallel means $\mathbf{b} = \lambda\mathbf{a}$ for one number λ. The $\mathbf{i}$ components give λ.

B5 $\mathbf{u} = -4\mathbf{i} + 6\mathbf{j} + p\mathbf{k}$ and $\mathbf{v} = q\mathbf{i} - 9\mathbf{j} + 3\mathbf{k}$ are parallel. Find p and q. [4 marks]

Answer .....

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C1 Lena finds a unit vector in the direction of $6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$. She writes: "$6 + 2 + 3 = 11$, so the unit vector is $\frac{1}{11}(6\mathbf{i} - 2\mathbf{j} + 3\mathbf{k})$". Find her mistake and give the correct unit vector. [4 marks]

Answer .....

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C2 $\overrightarrow{OA} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\overrightarrow{OB} = 5\mathbf{i} - \mathbf{j} + 3\mathbf{k}$. Tom writes "$\overrightarrow{AB} = \overrightarrow{OA} - \overrightarrow{OB} = -2\mathbf{i} + 3\mathbf{j} - 4\mathbf{k}$". Find his mistake, give the correct $\overrightarrow{AB}$, and find the angle $\overrightarrow{AB}$ makes with the positive y-axis, to 1 decimal place. [5 marks]

Answer .....

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C3 The vector $\mathbf{a} = t\mathbf{i} + 2\mathbf{j} - 2t\mathbf{k}$ has magnitude 7. Find the possible values of t. For the positive value, find the angle $\mathbf{a}$ makes with the positive x-axis, to 1 decimal place. [5 marks]

Answer .....

C4 The vector $2\mathbf{i} + 3\mathbf{j} + 6\mathbf{k}$ makes angles α, β and γ with the positive x-, y- and z-axes. Show that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$, and find γ to 1 decimal place. [4 marks]

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C5 A, B and C are the points $(1, 1, 1)$, $(3, 3, 2)$ and $(2, -1, 3)$. Show that triangle ABC is right-angled and isosceles, and find its area. [5 marks]

Answer .....

C6 Josh says that $\mathbf{a} = 2\mathbf{i} - 4\mathbf{j} + 6\mathbf{k}$ and $\mathbf{b} = -3\mathbf{i} + 6\mathbf{j} - 8\mathbf{k}$ are parallel, because the first two components of $\mathbf{b}$ are $-\frac{3}{2}$ times those of $\mathbf{a}$. Is he right? Show your working. [4 marks]

Answer .....

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D1 The points A, B and C have coordinates $(2, -1, 3)$, $(5, 3, 3)$ and $(-1, 3, 8)$.

(a) Find $\overrightarrow{AB}$ and $\overrightarrow{AC}$. [2 marks]

Answer .....

(b) Find angle BAC, to 1 decimal place. [4 marks]

Answer .....

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(c) Hence find the area of triangle ABC, to 3 significant figures. [2 marks]

Answer .....

(d) Find a unit vector in the direction of $\overrightarrow{BC}$. [2 marks]

Answer .....

Total for D1: 10 marks

D2

Kai finds the angle between $\mathbf{a} = 4\mathbf{i} - 4\mathbf{j} + 7\mathbf{k}$ and the positive y-axis.

He writes: "$|\mathbf{a}| = 9$ and $\cos\theta = \frac{4}{9}$, so $\theta = 63.6°$".

Kai is wrong.

(a) Show that $|\mathbf{a}| = 9$. [2 marks]

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(b) Explain Kai's mistake and find the correct angle, to 1 decimal place. [3 marks]

Answer .....

(c) Find the angle between $\mathbf{a}$ and the positive z-axis, to 1 decimal place. [2 marks]

Answer .....

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(d) The point P is such that $\overrightarrow{OP}$ is in the same direction as $\mathbf{a}$ and $OP = 27$. Find the coordinates of P. [2 marks]

Answer .....

Total for D2: 9 marks

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D3 The points P and Q lie on the z-axis, and each is 7 units from the point $A(2, 3, 1)$. Find the exact area of triangle APQ. [8 marks]

Answer .....

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D4 $\overrightarrow{OA} = 2\mathbf{i} + 5\mathbf{j} - \mathbf{k}$ and $\overrightarrow{OB} = 6\mathbf{i} + \mathbf{j} + 7\mathbf{k}$. M is the midpoint of AB.

(a) Find $\overrightarrow{AB}$ and the exact length AB. [3 marks]

Answer .....

(b) Using your answer to (a), find the position vector of M. [2 marks]

Answer .....

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(c) Hence find OM and the angle $\overrightarrow{OM}$ makes with the positive x-axis, to 1 decimal place. [3 marks]

Answer .....

(d) The point N lies on AB with $AN : NB = 1 : 3$. Find the coordinates of N. [2 marks]

Answer .....

Total for D4: 10 marks

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D5 $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ and $\mathbf{b} = -\mathbf{i} + 4\mathbf{j} + 2\mathbf{k}$.

(a) Find the values of p and q such that $p\mathbf{a} + q\mathbf{b} = 11\mathbf{i} - 2\mathbf{j} - 7\mathbf{k}$. [4 marks]

Answer .....

(b) Find the value of r for which $\mathbf{i} + 10\mathbf{j} + r\mathbf{k}$ can be written as $p\mathbf{a} + q\mathbf{b}$. [3 marks]

Answer .....

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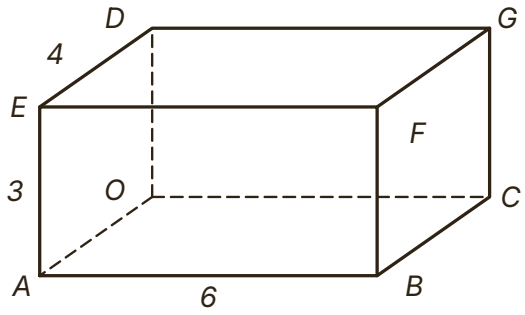
(c) Hence find the magnitude of $\mathbf{a} + 2\mathbf{b}$, to 3 significant figures. [2 marks]

Answer .....

Total for D5: 9 marks

D6 The diagram shows a cuboid $OABCDEFG$. O is the origin, A lies on the x-axis, C on the y-axis and D on the z-axis. $OA = 4$, $OC = 6$ and $OD = 3$.

(a) Write down the coordinates of F and G. [2 marks]



Answer .....

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(b) Find the exact length of the diagonal OF. [2 marks]

Answer .....

(c) Find the angle between OF and the z-axis, to 1 decimal place. [2 marks]

Answer .....

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(d) The diagonals OF and AG meet at M. Find the acute angle between them, to 1 decimal place. [4 marks]

Answer .....

Total for D6: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The point P lies on the line through O in the direction of $2\mathbf{i} + \mathbf{j} + 2\mathbf{k}$. The point A is $(5, 0, 4)$. Find the coordinates of the point P that is nearest to A, the least distance AP, and the exact area of triangle OAP for this P. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Any point on the line is $(2t, t, 2t)$ for some number t.

Hint 2 Write AP^2 as a quadratic in t. Its least value gives the nearest point.

Hint 3 At the nearest point, AP is at right angles to OP, so AP is the height of the triangle on the base OP.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the distance between two points in three dimensions.	○ ○ ○
I can find the magnitude of a 3D vector, a unit vector in its direction and the angle it makes with an axis.	○ ○ ○
I can find the sides, angles and area of a triangle from the coordinates of its corners.	○ ○ ○
I can use parallel vectors and given lengths to find unknown values.	○ ○ ○

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