Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-addition-formulae/

Addition formulae

114 marks

Date

A Do Now

Quick questions to start.

A1 Needed today [3 marks]

Write down the exact values of $\sin 30°$, $\cos 45°$ and $\tan 60°$.

Answer .....

A2 Needed today [3 marks]

$\sin\theta = \frac{3}{5}$ and θ is obtuse. Find the exact values of $\cos\theta$ and $\tan\theta$.

Answer .....

A3 Common slip [2 marks]

Show that $\sin(30° + 60°)$ is not equal to $\sin 30° + \sin 60°$.

A4 From chapter 6 [2 marks]

Solve $\tan x = -1$ for $0° \leq x < 360°$.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Use $\cos(A+B) = \cos A \cos B - \sin A \sin B$ to find the exact value of $\cos 75^\circ$.

Hint Write 75° as a sum of two angles whose sines and cosines you know.

B1 Find the exact values of $\sin 15^\circ$ and $\cos 105^\circ$. [4 marks]

Answer .....

EXAMPLE 2

Find the exact value of $\tan 15^\circ$, in the form $a + b\sqrt{3}$.

Hint Use tan(A − B) = (tan A − tan B)/(1 + tan A tan B). The sign in the bottom is the opposite of the sign on top.

B2 Find the exact value of $\tan 105^\circ$, in the form $a + b\sqrt{3}$. [4 marks]

Answer .....

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EXAMPLE 3

$\sin A = \frac{4}{5}$ with A acute, and $\cos B = -\frac{5}{13}$ with B obtuse. Find the exact values of $\sin(A + B)$ and $\cos(A - B)$.

Hint Find cos A and sin B first. In the second quadrant sine is positive.

B3 [6 marks]

$\sin A = \frac{8}{17}$ with A acute, and $\tan B = -\frac{3}{4}$ with $90° < B < 180°$. Find the exact values of $\cos(A + B)$ and $\tan(A - B)$.

Answer .....

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EXAMPLE 4

Solve $\sin(x + 30^\circ) = 2\cos x$ for $0^\circ \leq x < 360^\circ$.

Hint Expand the left side, then collect the sin x terms on one side and the cos x terms on the other.

B4 Solve $\cos(x - 60^\circ) = \sqrt{3}\sin x$ for $0^\circ \leq x < 360^\circ$. [5 marks]

Answer .....

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EXAMPLE 5

Show that $\sin(A+B)+\sin(A-B)=2\sin A\cos B$. Hence find the exact value of $\sin 75^\circ \cos 15^\circ$.

Hint Expand both, and the cos A sin B terms cancel. Then choose A and B so that A + B and A − B are angles you know.

B5 Prove that $\cos(A+B)\cos(A-B)$ is equal to $\cos^2 A-\sin^2 B$. Hence find the exact value of $\cos^2 75^\circ-\sin^2 45^\circ$. [6 marks]

Answer .....

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C1 Mia works out $\tan 75^\circ$ like this: [4 marks]

$$\tan 75^\circ = \tan(45^\circ + 30^\circ) = \frac{1 + \frac{1}{\sqrt{3}}}{1 + \frac{1}{\sqrt{3}}} = 1.$$

Explain the mistake, and find the exact value of $\tan 75^\circ$ in the form $a + b\sqrt{3}$.

Answer .....

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C2 Write each as a single trigonometric function: (i) $\cos 5x \cos 2x + \sin 5x \sin 2x$ (ii) $\dfrac{\tan 3\theta + \tan \theta}{1 - \tan 3\theta \tan \theta}$. Hence find the exact value of $\cos \dfrac{5\pi}{12} \cos \dfrac{\pi}{12} + \sin \dfrac{5\pi}{12} \sin \dfrac{\pi}{12}$. [4 marks]

Answer .....

C3 $\tan(x + y) = 3$ and $\tan x = \dfrac{1}{2}$. Find the exact value of $\tan y$. [4 marks]

Answer .....

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C4 Show that $\sin\left(x+\frac{\pi}{6}\right)-\sin\left(x-\frac{\pi}{6}\right)=\cos x$. Hence solve $2\sin\left(x+\frac{\pi}{6}\right)-2\sin\left(x-\frac{\pi}{6}\right)=\sqrt{3}$ for $0\leq x<2\pi$. [5 marks]

C5 A and B are acute angles with $\tan A=\frac{1}{2}$ and $\tan B=\frac{1}{3}$. Show that $A+B=\frac{\pi}{4}$. [4 marks]

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D1

(a) Show that $\sin(x + 30^\circ) + \cos(x + 60^\circ) = \cos x$. [3 marks]

(b) Hence solve $\sin(2\theta + 30^\circ) + \cos(2\theta + 60^\circ) = -\frac{1}{2}$ for $0^\circ \leq \theta < 360^\circ$. [4 marks]

Answer .....

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(c) Hence find the exact value of $\sin 105° + \cos 135°$. [2 marks]

Answer .....

Total for D1: 9 marks

D2

> Sam writes: "$\tan(A + B) = \dfrac{3 + 2}{1 - 6} = -1$, so $A + B = -45°$. That is impossible for two acute angles, so no such angles exist."

A and B are acute angles with $\tan A = 3$ and $\tan B = 2$.

(a) Explain Sam's mistake, and state the size of $A + B$. [2 marks]

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(b) Find the exact value of $\cos(A - B)$, with a rational denominator. [4 marks]

Answer .....

(c) Find the exact value of $\tan(A - B)$. [2 marks]

Answer .....

Total for D2: 8 marks

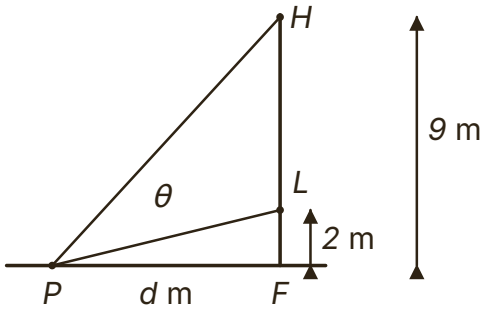
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D3 Solve $2\sin\left(x+\frac{\pi}{4}\right)=\cos\left(x-\frac{\pi}{4}\right)$ for $0\leq x<2\pi$. [6 marks]

Answer .....

D4 A wall stands on level ground. Point P is on the ground, d metres from the foot F of the wall. Marks L and H are on the wall, 2 m and 9 m above F. The angle LPH is θ.

(a) When $d=6$, show that $\tan\theta=\frac{7}{9}$. [3 marks]



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(b) Show that $\tan\theta = \dfrac{7d}{d^2 + 18}$ for any $d > 0$. [3 marks]

(c) Hence find the two distances d for which $\tan\theta = \dfrac{7}{11}$. [3 marks]

Answer .....

Total for D4: 9 marks

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D5

(a) Given that $\sin(x+\alpha) = 2\sin(x-\alpha)$, where $\cos x \cos\alpha \neq 0$, show that $\tan x = 3\tan\alpha$. [4 marks]

(b) Hence solve $\sin\left(x+\frac{\pi}{6}\right) = 2\sin\left(x-\frac{\pi}{6}\right)$ for $0 \leq x < 2\pi$. [3 marks]

Answer .....

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(c) Solve $\sin\left(x + \frac{\pi}{2}\right) = 2\sin\left(x - \frac{\pi}{2}\right)$ for $0 \leq x < 2\pi$, explaining why part (a) cannot be used. [2 marks]

Answer .....

Total for D5: 9 marks

D6

(a) Prove that $\cos(A + B) + \cos(A - B) = 2\cos A \cos B$. [2 marks]

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(b) Hence solve $\cos 3x + \cos x = 0$ for $0 \leq x \leq \pi$. [4 marks]

Answer .....

(c) Hence find the exact value of $\cos 75^\circ + \cos 15^\circ$. [3 marks]

Answer .....

Total for D6: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The three angles of a triangle are A, B and C, and $\tan A$, $\tan B$ and $\tan C$ are all positive whole numbers. Find them. [8 marks]

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Since $A + B + C = 180°$, $\tan C = -\tan(A + B)$. Expand and clear the fraction.

Hint 2 You should reach $\tan A + \tan B + \tan C = \tan A \tan B \tan C$. Call the three whole numbers $x \le y \le z$.

Hint 3 $xyz = x + y + z \le 3z$, so $xy \le 3$. Check each small case.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use sin(A ± B), cos(A ± B) and tan(A ± B) to find exact values such as cos 75° and tan 15°.	○ ○ ○
I can find sin(A + B) or cos(A − B) from given ratios, choosing the signs from the quadrant.	○ ○ ○
I can write an expression such as cos 5x cos 2x + sin 5x sin 2x as a single trigonometric function.	○ ○ ○
I can expand both sides of an equation to solve it, and prove results that use the addition formulae.	○ ○ ○

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