Algebraic division and improper fractions

111 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-algebraic-division-and-improper-fractions/

Date

A Do Now

Quick questions to start.

A1 Needed today [3 marks]

Divide $x^3 + 2x^2 - 5x + 6$ by $(x + 4)$. Give the quotient and the remainder.

Answer .....

A2 Needed today [2 marks]

Is $\dfrac{x^2 + 1}{x^2 - 1}$ a proper or an improper fraction? Give a reason.

Answer .....

A3 Last lesson [3 marks]

Express $\dfrac{5x - 1}{(x + 1)(x - 2)}$ in partial fractions.

Answer .....

A4 Needed today [2 marks]

Find A and B such that $\dfrac{2x + 5}{x + 1} \equiv A + \dfrac{B}{x + 1}$.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Express $\dfrac{3x^2 - 2x - 2}{x^2 - x}$ in partial fractions.

Hint The top and bottom have the same degree, so start with a whole number: $\dfrac{3x^2 - 2x - 2}{x(x-1)} \equiv A + \dfrac{B}{x} + \dfrac{C}{x-1}$.

B1 Express $\dfrac{2x^2 - x + 5}{(x+1)(x-3)}$ in partial fractions. [5 marks]

Answer .....

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EXAMPLE 2

Express $\dfrac{x^3 + x^2 + 2x - 2}{x^2 - 1}$ in partial fractions.

Hint The top has degree 3 and the bottom degree 2, so divide first. The quotient is linear.

B2 Express $\dfrac{2x^3 - 3x^2 - 5x + 9}{x^2 - x - 2}$ in partial fractions. [6 marks]

Answer .....

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EXAMPLE 3

Express $\dfrac{4x^2+x}{(2x-1)(x+1)}$ in partial fractions.

Hint Multiply out the bottom: $2x^2+x-1$. The whole number part is $\dfrac{4}{2}=2$, not 4.

B3 Express $\dfrac{9x^2+14x-1}{(3x-1)(x+2)}$ in partial fractions. [5 marks]

Answer .....

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EXAMPLE 4

Express $\dfrac{x^3 - 2x^2 + 4}{x^2(x-2)}$ in partial fractions.

Hint Same degree, and a repeated factor x^2: use $A + \dfrac{B}{x} + \dfrac{C}{x^2} + \dfrac{D}{x-2}$.

B4 Express $\dfrac{2x^3 - x + 5}{(x-1)^2(x+2)}$ in partial fractions. [6 marks]

Answer .....

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C1 Raj says: "$\dfrac{x^2+4x-1}{(x-1)(x+3)} \equiv \dfrac{1}{x-1}+\dfrac{1}{x+3}$." Explain the mistake, give the correct partial fractions, and check your answer at $x=0$. [5 marks]

Answer .....

C2 Write $\dfrac{x^3+3x^2-2}{x+2}$ in the form $ax^2+bx+c+\dfrac{d}{x+2}$. [4 marks]

Answer .....

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C3 Given that $\dfrac{x^3 + ax^2 + 2}{x^2 + x - 2} \equiv x + 2 + \dfrac{B}{x - 1} + \dfrac{C}{x + 2}$, find a, B and C. [5 marks]

Answer $a = \ldots$, $B = \ldots$, $C = \ldots$

C4 Lucy says that $\dfrac{3x^2 + 1}{x^2 + x}$ is a proper fraction because the bottom has two terms and the top has two terms. Explain why she is wrong, then express the fraction in partial fractions. [5 marks]

Answer .....

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D Exam-style questions

Edexcel exam style

D1 Express $\dfrac{4x^3 - 4x + 10}{2x^2 + x - 3}$ in partial fractions. [7 marks]

Answer .....

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D2 $f(x) = 2x^3 + 3x^2 - 3x - 2.$

(a) Show that $(x - 1)$ is a factor of $f(x)$ and hence factorise $f(x)$ fully. [3 marks]

Answer .....

(b) Hence express $\dfrac{4x^3 + 8x^2 + 2x - 5}{f(x)}$ in partial fractions. [6 marks]

Answer .....

Total for D2: 9 marks

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D3

Zoe writes $\dfrac{x^3+1}{x^2-x} = x + \dfrac{3}{x}$.

(a) By substituting $x = 2$, show that Zoe's final answer is wrong. [2 marks]

(b) Express $\dfrac{x^3+1}{x^2-x}$ correctly in partial fractions. [6 marks]

Answer .....

Total for D3: 8 marks

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D4 $f(x) = \dfrac{2x^2 + 7x + 7}{(x+1)(x+2)}$, $x \geq 0$.

(a) Express $f(x)$ in the form $A + \dfrac{B}{x+1} + \dfrac{C}{x+2}$. [5 marks]

Answer $A = \ldots$, $B = \ldots$, $C = \ldots$

(b) Hence show that $\displaystyle\int_0^2 f(x)\,dx = 4 + \ln\frac{9}{2}$. [6 marks]

Total for D4: 11 marks

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D5 $f(x) = \dfrac{2x^2 + 5x + 5}{x + 2}$, $x \neq -2$.

(a) Write $f(x)$ in the form $ax + b + \dfrac{c}{x + 2}$. [3 marks]

Answer .....

(b) Show that $f(x) > 2x + 1$ for all $x > -2$. [3 marks]

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(c) Solve $f(x) = x + 4$. Give your answers in surd form. [3 marks]

Answer .....

Total for D5: 9 marks

D6 Given that $\dfrac{4x^3 + px^2 + qx + 5}{(x-1)(x+3)} \equiv 4x + 1 + \dfrac{3}{x-1} + \dfrac{R}{x+3}$, find the values of p, q and R. [7 marks]

Answer $p = \ldots$, $q = \ldots$, $R = \ldots$

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Find all the integers x for which $\dfrac{x^4 + 2x^3 + 3}{x^2 + x - 2}$ is an integer. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Divide $x^4 + 2x^3 + 3$ by $x^2 + x - 2$. The quotient is always an integer when x is.

Hint 2 The remainder is $x + 5$. Split $\dfrac{x+5}{(x-1)(x+2)}$ into partial fractions.

Hint 3 When x is large, $\dfrac{2}{x-1} - \dfrac{1}{x+2}$ is a small number that is not zero. Check the small integers one by one.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can tell whether an algebraic fraction is improper by comparing degrees.	○ ○ ○
I can divide polynomials to find the quotient and the remainder.	○ ○ ○
I can split an improper fraction into a polynomial part and partial fractions.	○ ○ ○
I can find unknown constants in an identity by comparing coefficients.	○ ○ ○

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