

Algebraic fractions and dividing polynomials

73 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-algebraic-fractions-and-dividing-polys/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] Factorise $5x^2 - 13x - 6$.
- A2** Needed today [2 marks] Work out $9437 \div 26$, giving the remainder.

Answer

Answer remainder

- A3** Last lesson [2 marks] Find the tangent to $x^2 + y^2 = 13$ at $(2, -3)$.
- A4** From chapter 1 [2 marks] Simplify $(12x^5 - 8x^3) \div 4x^2$.

Answer

Answer

B Example, then your turn

A-Level

Work through each example, then try the one beside it.

EXAMPLE 1

Simplify fully $\frac{3x^2 + 7x - 6}{x^2 - 9}$.

Hint Factorise the top and the bottom. You may only cancel a bracket that is a factor of the WHOLE top and the WHOLE bottom.

B1 Simplify fully $\frac{2x^2 - 3x - 20}{4x^2 - 25}$. [3 marks]

Answer

EXAMPLE 2

Divide $2x^3 - 3x^2 - 11x + 6$ by $(x + 2)$, and hence factorise $2x^3 - 3x^2 - 11x + 6$ completely.

Hint Divide the first term by x , multiply $(x + 2)$ by what you get, subtract, bring down the next term, and repeat.

B2

Divide

[5 marks]

$3x^3 + 8x^2 - 15x + 4$ by $(x + 4)$. Hence solve $3x^3 + 8x^2 - 15x + 4 = 0$.

Answer $x = \dots\dots$, $x = \dots\dots$, $x = \dots\dots$

EXAMPLE 3

Find the quotient and the remainder when $4x^3 - 10x^2 + 5$ is divided by $(2x + 1)$.

Hint There is no x term: write $4x^3 - 10x^2 + 0x + 5$ so that every power has its own column. Each step divides by $2x$.

B3

Find the quotient and the remainder when $6x^4 - 5x^3 + 10x^2 + 3$ is divided by $(3x - 1)$.

[5 marks]

Answer quotient $\dots\dots$, remainder $\dots\dots$

- C1** Use algebraic division to show that $(x + 3)$ is a factor of $3x^3 + x^2 - 20x + 12$. Hence solve $3x^3 + x^2 - 20x + 12 = 0$. [5 marks]

Answer $x = \dots\dots$, $x = \dots\dots$, $x = \dots\dots$

- C2** Given that $(x + 4)$ is a factor of $x^3 + 3x^2 - 6x - 8$, simplify fully $\frac{x^3 + 3x^2 - 6x - 8}{x^2 + 6x + 8}$. [6 marks]

Answer

C3 Divide $8x^3 + 27$ by $(2x + 3)$.

[3 marks]

Answer

D Exam-style questions

Edexcel exam style

D1 Simplify fully $\frac{x^2 + 5x - 24}{x^2 - 9}$. Hence solve the equation $\frac{x^2 + 5x - 24}{x^2 - 9} = 3$.

[6 marks]

Answer ; $x = \dots\dots$

D2 Priya simplifies $\frac{4x^3 + 2x^2 - 42x}{2x^2 + 9x + 7}$. She writes:

$\frac{4x^3 + 2x^2 - 42x}{2x^2 + 9x + 7} = \frac{4x^3 - 42x}{9x + 7}$, because the $2x^2$ on the top and the $2x^2$ on the bottom cancel.

Priya is wrong.

(a) Explain the mistake Priya has made.

[1 mark]

- (b) The fraction can be written as $\frac{ax(x+b)}{x+c}$, where a , b and c are integers. Find the values of a , b and c . [4 marks]

Answer $a = \dots$, $b = \dots$, $c = \dots$

- (c) Hence show that there is no value of x for which $\frac{4x^3 + 2x^2 - 42x}{2x^2 + 9x + 7} = -8$. [3 marks]

Total for D2: 8 marks

D3 $f(x) = 2x^3 - 7x^2 - 10x + 24$

- (a) Given that $(x + 2)$ is a factor of $f(x)$, write $f(x)$ in the form $(x + 2)(ax^2 + bx + c)$, where a , b and c are integers. [3 marks]

Answer

(b) Hence factorise $f(x)$ completely.

[2 marks]

Answer

(c) Hence solve $2y^6 - 7y^4 - 10y^2 + 24 = 0$, giving your answers in exact form. [4 marks]

Answer $y = \dots\dots$

Total for D3: 9 marks

D4 $f(x) = 2x^3 + 3x^2 + 5x - 10$

(a) Use algebraic division to find the remainder when $f(x)$ is divided by $(2x + 1)$. [3 marks]

Answer remainder

- (b) Show that $(x - 1)$ is a factor of $f(x)$, and write $f(x)$ in the form $(x - 1)(ax^2 + bx + c)$. [3 marks]

Answer

- (c) Hence show that the equation $f(x) = 0$ has exactly one real root. [2 marks]

Total for D4: 8 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** When the polynomial $p(x)$ is divided by $(x - 1)$ the remainder is 3, [7 marks]
and when $p(x)$ is divided by $(x - 2)$ the remainder is 5. Find the
remainder when $p(x)$ is divided by $(x - 1)(x - 2)$.

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Dividing by $(x - 1)$ gives
 $p(x) = (x - 1)s(x) + 3$, so $p(1) = 3$. In the
same way $p(2) = 5$.
- Hint 2** Dividing by a quadratic leaves a remainder
with no x^2 term:
 $p(x) = (x - 1)(x - 2)q(x) + ax + b$.
- Hint 3** Put $x = 1$: $a + b = 3$. Put $x = 2$: $2a + b = 5$.
So $a = 2$ and $b = 1$: the remainder is $2x + 1$.

How did it go?

Colour one circle on each line: not yet, nearly,
yes.

I can simplify an algebraic
fraction by factorising and
cancelling common factors. ☐ ☐ ☐

I can divide a polynomial by a
linear expression, writing any
missing terms as zero. ☐ ☐ ☐

I can find the remainder, and
use division to factorise a
cubic completely. ☐ ☐ ☐

I can use the quotient to solve
a cubic or show how many
real roots it has. ☐ ☐ ☐