Algebraic fractions

112 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-algebraic-fractions-simplify-and-solve/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Factorise $2x^2 + 5x - 3$.

Answer .....

A2 Needed today [2 marks]

Factorise fully $x^3 - 9x$.

Answer .....

A3 Common slip [2 marks]

Jay says $\frac{x+6}{x+2} = \frac{6}{2} = 3$. Is he right? Give a reason.

A4 Needed today [2 marks]

Write $\frac{2}{x} + \frac{3}{x+1}$ as a single fraction.

Answer .....

A5 Common slip [2 marks]

Simplify $3(x+1) - (x-4)$.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Simplify fully $\dfrac{x^2-9}{2x^2+5x-3}$.

Hint Factorise the top and the bottom first. Only a factor that multiplies the whole top and the whole bottom can be cancelled.

B1 Simplify fully $\dfrac{3x^2-12}{x^3+2x^2}$. [3 marks]

Answer .....

EXAMPLE 2

Simplify fully $\dfrac{x^2+3x}{x^2-4} \div \dfrac{x^2-9}{x+2}$.

Hint Dividing by a fraction is the same as multiplying by it turned upside down. Then factorise everything and cancel.

B2 Simplify fully $\dfrac{2x^2-x-1}{x^2-1} \times \dfrac{x^2+2x+1}{4x^2-1}$. [4 marks]

Answer .....

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EXAMPLE 3

Write $\dfrac{3}{x+1} - \dfrac{x-5}{x^2-1}$ as a single fraction in its simplest form.

Hint $x^2 - 1 = (x-1)(x+1)$, so that is the common denominator. Put the subtracted top in a bracket.

B3 Write $\dfrac{2}{x-3} - \dfrac{x+9}{x^2-9}$ as a single fraction in its simplest form. [4 marks]

Answer .....

EXAMPLE 4

Simplify $\dfrac{1 - \frac{4}{x^2}}{1 + \frac{2}{x}}$.

Hint Multiply the top and the bottom of the big fraction by x^2. This clears the small fractions.

B4 Simplify $\dfrac{\frac{1}{x} - \frac{1}{3}}{9 - x^2}$. [4 marks]

Answer .....

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EXAMPLE 5

Solve $\frac{6}{x} - \frac{3}{x+2} = 1.$

Hint Multiply every term by $x(x+2)$. Then check that no answer makes a denominator zero.

B5 Solve $\frac{4}{x-1} + \frac{5}{x+2} = 3.$ [5 marks]

Answer $x =$ ..... or $x =$ .....

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C1 Ravi simplifies $\dfrac{x^2 + 5x}{x^2 + 3x - 10}$ by cancelling the x^2 terms, and gets $\dfrac{5x}{3x - 10}$. Show that Ravi is wrong, and find the correct answer. [4 marks]

Answer .....

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C2 Mia writes $\dfrac{2x}{x+3} - \dfrac{x-1}{x+3} = \dfrac{2x-x-1}{x+3} = \dfrac{x-1}{x+3}$. Explain her mistake. Then write $\dfrac{3x+1}{x^2-4} - \dfrac{x-1}{x^2-2x}$ as a single fraction in its simplest form. [5 marks]

Answer .....

C3 Simplify fully $\dfrac{x^2-16}{x^2+x-12} \div \dfrac{x^2-4x}{x^2-9}$. [4 marks]

Answer .....

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C4 [5 marks]

$f(x) = \dfrac{x^2 - 2x - 8}{x^2 - 16}$. Simplify $f(x)$, and hence show that the equation $f(x) = \dfrac{3}{4}$ has no solution.

Answer .....

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C5 Write $\dfrac{2x^2 + x - 6}{x^2 - 4} + \dfrac{x}{x + 2}$ as a single fraction in its simplest form. [5 marks]

Answer .....

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D Exam-style questions

Edexcel exam style

D1 Solve $\dfrac{3x}{x+2} - \dfrac{x+14}{x^2-x-6} = 2$, giving your answers in the form $a \pm b\sqrt{2}$, where a and b are integers. [7 marks]

Answer $x = \ldots\ldots$

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D2 $f(x) = \dfrac{x+1}{x-2} - \dfrac{2}{x^2-4}$, $x \neq \pm 2$.

(a) Show that $f(x) = \dfrac{x(x+3)}{x^2-4}$. [3 marks]

(b) Solve $f(x) = 1$. [3 marks]

Answer .....

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(c) Solve $f(x) = 2$, giving your answers in surd form. [3 marks]

Answer .....

Total for D2: 9 marks

D3

> Leo says: "$\dfrac{3}{x-2} - \dfrac{x+4}{x^2-4} = \dfrac{3-x-4}{x^2-4} = \dfrac{-x-1}{x^2-4}$."

Leo writes $\dfrac{3}{x-2} - \dfrac{x+4}{x^2-4}$ over the denominator $x^2 - 4$.

(a) By substituting $x = 0$, show that Leo is wrong. [2 marks]

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(b) Write $\dfrac{3}{x-2} - \dfrac{x+4}{x^2-4}$ correctly as a single fraction in its simplest form. [3 marks]

Answer .....

(c) Solve $\dfrac{3}{x-2} - \dfrac{x+4}{x^2-4} = \dfrac{1}{2}$, giving your answers in the form $a \pm b\sqrt{3}$. [3 marks]

Answer .....

Total for D3: 8 marks

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D4 The curve C has equation $y = \dfrac{x^2 + x - 6}{x^2 - 3x + 2}$.

(a) Show that $y = 1 + \dfrac{4}{x - 1}$, and state the two values of x for which y is not defined. [4 marks]

(b) The graph of $y = 1 + \dfrac{4}{x - 1}$ passes through a point that is missing from C. Find its coordinates. [3 marks]

Answer .....

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(c) Find the set of values of x for which $y > 3$ on C. [4 marks]

Answer .....

Total for D4: 11 marks

D5 $f(x) = \dfrac{1}{x} - \dfrac{2}{x+1} + \dfrac{1}{x+2}$, $x \neq 0, -1, -2$.

(a) Show that $f(x) = \dfrac{2}{x(x+1)(x+2)}$. [4 marks]

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(b) Hence solve $f(x) = \dfrac{1}{3x}$. [3 marks]

Answer .....

(c) Hence show that $f(x) < \dfrac{2}{x^3}$ for all $x > 0$. [2 marks]

Total for D5: 9 marks

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D6 Given that $\dfrac{x^2+ax+b}{x^2-5x+6}$ simplifies to $\dfrac{x+4}{x-3}$, find the values of the constants a and b. [6 marks]

Answer $a = \ldots\ldots$, $b = \ldots\ldots$

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Find all the pairs of positive integers x and y such that $\frac{1}{x} + \frac{1}{y} = \frac{1}{4}$. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Multiply through by $4xy$ to clear the fractions.

Hint 2 You get $xy - 4x - 4y = 0$. Add 16 to both sides and factorise the left side.

Hint 3 $(x-4)(y-4) = 16$. Why must both brackets be positive? Then list the ways to write 16 as a product.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can simplify an algebraic fraction by cancelling common factors.	○ ○ ○
I can multiply, divide, add and subtract algebraic fractions.	○ ○ ○
I can solve equations with algebraic fractions and check for values that are not allowed.	○ ○ ○
I can use a single fraction to solve an equation or show an inequality.	○ ○ ○

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