

## Angles in all four quadrants and exact values

70 marks



Hints online, answers free  
with an account:  
[mathswariz.uk/worksheets/a-level-angles-in-all-four-quadrants-and-exact-values/](https://mathswariz.uk/worksheets/a-level-angles-in-all-four-quadrants-and-exact-values/)

Date \_\_\_\_\_

### **A** Do Now

Quick questions to start.

- |                                                                                                                                                            |                                                                                                                                    |
|------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>A1</b> [Last lesson] [2 marks]</p> <p>Write down the period of <math>y = \cos 3x</math> and the maximum value of <math>y = 4 - 2 \cos 3x</math>.</p> | <p><b>A2</b> [Needed today] [2 marks]</p> <p>An equilateral triangle has sides of 2 cm. Find the exact height of the triangle.</p> |
|------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------|

Answer ..... and .....

Answer .....

- |                                                                                                                                             |                                                                                                                                                                                                               |
|---------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <p><b>A3</b> [From chapter 1] [2 marks]</p> <p>Write <math>\frac{3}{\sqrt{6}}</math> with a rational denominator, in its simplest form.</p> | <p><b>A4</b> [From chapter 5] [2 marks]</p> <p><math>O</math> is the origin and <math>P</math> is the point <math>(-3, 4)</math>. Find the gradient of <math>OP</math> and the length of <math>OP</math>.</p> |
|---------------------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

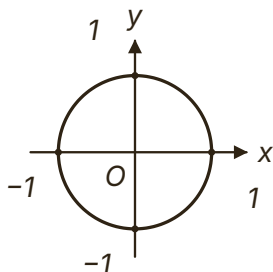
Answer .....

Answer ..... and .....

Your teacher works through each example with you. Then try the one beside it.

**EXAMPLE 1**

Use the unit circle to write down the values of  $\sin 630^\circ$ ,  $\cos(-540^\circ)$  and  $\tan 900^\circ$ .



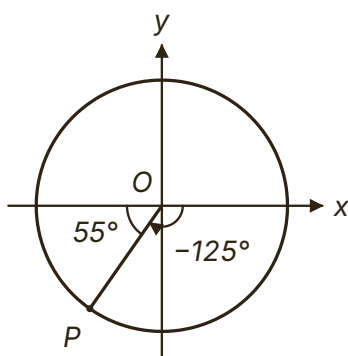
**Hint** Bring each angle into  $0^\circ$  to  $360^\circ$ , then read the circle.

**B1** Without a calculator, [3 marks]  
write down the values  
of  $\cos 990^\circ$ ,  $\sin(-630^\circ)$   
and  $\tan 1125^\circ$ .

Answer ..... , ..... , .....

**EXAMPLE 2**

Write  $\sin 290^\circ$ ,  $\cos(-125^\circ)$  and  $\tan 470^\circ$  as ratios of acute angles.



**Hint** Acute angle to the  $x$ -axis, then the sign.

**B2** Express  $\cos 215^\circ$ , [3 marks]  
 $\tan(-320^\circ)$  and  
 $\sin 685^\circ$  in terms of  
trigonometric ratios of  
acute angles.

Answer ..... , ..... , .....

**EXAMPLE 3**

$\theta$  is an acute angle. Express  $\sin(900^\circ - \theta)$ ,  $\cos(-180^\circ - \theta)$  and  $\tan(\theta - 540^\circ)$  in terms of  $\sin \theta$ ,  $\cos \theta$  or  $\tan \theta$ .

**Hint** Take away whole turns of  $360^\circ$  first. Then draw the angle and decide its quadrant.

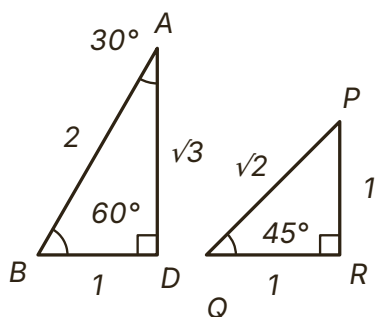
**B3**  $\theta$  is an acute angle. [3 marks]

Express  $\cos(720^\circ - \theta)$ ,  $\sin(-\theta - 180^\circ)$  and  $\tan(-\theta - 900^\circ)$  in terms of  $\sin \theta$ ,  $\cos \theta$  or  $\tan \theta$ .

Answer ..... , ..... , .....

**EXAMPLE 4**

Find the exact values of  $\cos 315^\circ$ ,  $\sin(-240^\circ)$  and  $\tan 510^\circ$ .



**Hint** The two triangles give the exact values for  $30^\circ$ ,  $45^\circ$  and  $60^\circ$ . Find the acute angle, then the sign.

**B4** Find the exact values of [4 marks]

$\cos(-150^\circ)$ ,  $\sin 675^\circ$ ,  $\tan 240^\circ$  and  $\cos 840^\circ$ .

- C1** For each angle, state the quadrant that  $OP$  lies in, and whether  $\sin \theta$ ,  $\cos \theta$  and  $\tan \theta$  are positive or negative. (a)  $\theta = 500^\circ$  (b)  $\theta = -290^\circ$  [4 marks]

- C2** Show that  $\cos 210^\circ \sin 240^\circ + \sin 330^\circ \cos(-300^\circ) = \frac{1}{2}$ . [4 marks]

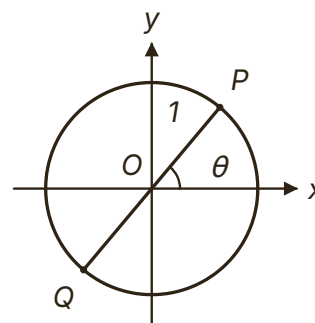
- C3**  $\sin \theta < 0$  and  $\tan \theta > 0$ , where  $0^\circ < \theta < 360^\circ$ . State the quadrant that  $\theta$  is in. Given also that  $OP$  makes an acute angle of  $25^\circ$  with the  $x$ -axis, find  $\theta$ . [4 marks]

Answer ..... quadrant  $\theta = \dots\dots$

**D1** The diagram shows the unit circle.  $OP$  makes an acute angle  $\theta$  with the positive  $x$ -axis, and  $OQ$  makes an angle of  $180^\circ + \theta$ .

- (a)** Write down the coordinates of  $P$ . [3 marks]

Explain why  $Q$  has coordinates  $(-\cos \theta, -\sin \theta)$ .



- (b)** Hence show that  $\sin(180^\circ + \theta) = -\sin \theta$  and  $\tan(180^\circ + \theta) = \tan \theta$ . [3 marks]

- (c)** Hence find the exact value of  $\sin 225^\circ + \tan 240^\circ$ . Give your answer as a single fraction. [4 marks]

Answer .....

**Total for D1: 10 marks**

**D2**

Sam writes: " $330^\circ$  and  $30^\circ$  both make an angle of  $30^\circ$  with the  $x$ -axis, so

$$\tan 330^\circ = \tan 30^\circ = \frac{\sqrt{3}}{3}."$$

Sam is wrong.

- (a) Explain Sam's mistake, and give the exact value of  $\tan 330^\circ$ . [2 marks]

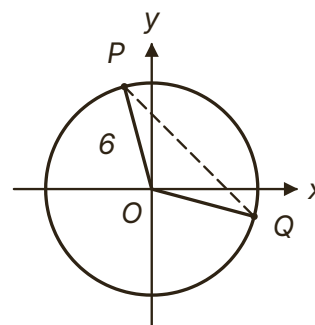
- (b) Find the exact value of  $\frac{\tan 330^\circ \cos(-420^\circ)}{\sin 480^\circ}$ . [4 marks]

Answer .....

Total for D2: 6 marks

- D3** The points  $P$  and  $Q$  lie on a circle with centre  $O(0,0)$  and radius 6.  $OP$  makes an angle of  $120^\circ$  with the positive  $x$ -axis and  $OQ$  makes an angle of  $330^\circ$ .

- (a) Find the exact coordinates of  $P$  and of  $Q$ . [4 marks]



Not drawn accurately

(b) Hence find the exact area of triangle  $OPQ$ .

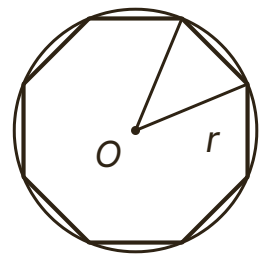
[3 marks]

Answer .....

Total for D3: 7 marks

**D4** A regular octagon has its eight corners on a circle with centre  $O$  and radius  $r$  cm.

(a) Show that the area of the octagon is  $2\sqrt{2}r^2$  cm<sup>2</sup>. [3 marks]



(b) The area of the octagon is  $98\sqrt{2}$  cm<sup>2</sup>. Hence find  $r$ .

[2 marks]

Answer  $r = \dots\dots$

(c) Find the exact perimeter of the octagon. Give it in the form  $a\sqrt{2 - \sqrt{2}}$ , where  $a$  is an integer. [3 marks]

Answer .....

Total for D4: 8 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Without a calculator, find the exact value of  $\tan 22.5^\circ$  in the form  $a + b\sqrt{2}$ . Hence find the exact value of  $\tan 202.5^\circ \times \tan 157.5^\circ$ . [6 marks]

Answer ..... and .....

## Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Start with a right-angled triangle  $ABC$  with  $AB = BC = 1$  and the right angle at  $B$ , so angle  $ACB = 45^\circ$ . Halve that  $45^\circ$  with an isosceles triangle.
- Hint 2** Extend  $BC$  past  $C$  to  $D$  so that  $CD = CA = \sqrt{2}$ . Triangle  $ACD$  is isosceles and angle  $ACD = 135^\circ$ , so angle  $ADC = 22.5^\circ$ .
- Hint 3**  $\tan 22.5^\circ = \frac{AB}{BD} = \frac{1}{1 + \sqrt{2}} = \sqrt{2} - 1$ .  
 $\tan 202.5^\circ = \tan 22.5^\circ$  and  
 $\tan 157.5^\circ = -\tan 22.5^\circ$ , so the product is  
 $-(\sqrt{2} - 1)^2 = 2\sqrt{2} - 3$

## How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use the unit circle to find sin, cos and tan of any angle. ☐ ☐ ☐

I can say whether sin, cos and tan are positive or negative in each quadrant. ☐ ☐ ☐

I can write down the exact values for  $30^\circ$ ,  $45^\circ$  and  $60^\circ$ . ☐ ☐ ☐

I can find exact values for related angles of any size. ☐ ☐ ☐