

Areas under the x-axis and between curves and lines

70 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-areas-below-and-between-curves/

Date _____

A Do Now

Quick questions to start.

- | | |
|--|--|
| <p>A1 Needed today [2 marks]</p> <p>Solve $x^2 - 3x = x + 5$.</p> | <p>A2 Last lesson [2 marks]</p> <p>Find the exact value of $\int_1^3 (x^2 - 1) dx$.</p> |
|--|--|

Answer $x = \dots\dots$

Answer

- | | |
|--|--|
| <p>A3 From chapter 7 [2 marks]</p> <p>Factorise $x^3 - 2x^2 - 5x + 6$ completely.</p> | <p>A4 From chapter 5 [2 marks]</p> <p>Find the area of the triangle between $y = 12 - 3x$ and the axes.</p> |
|--|--|

Answer

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find the area of the finite region bounded by the curve $y = x^2 - 4x$ and the x -axis.

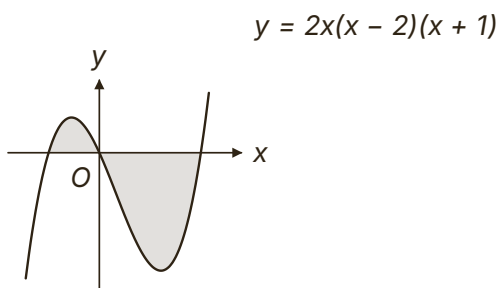
Hint Between its roots the curve is below the axis, so the integral is negative.

- B1** Find the area of the [2 marks]
finite region bounded
by the curve
 $y = 3x^2 - 12$ and the x -
axis.

Answer

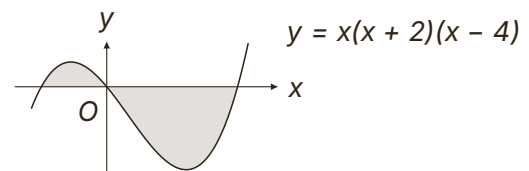
EXAMPLE 2

The sketch shows the curve $y = 2x(x - 2)(x + 1)$. Find the total area of the two shaded regions.



Hint Integrate from -1 to 0 and from 0 to 2 separately. One part is below the axis, so take its size.

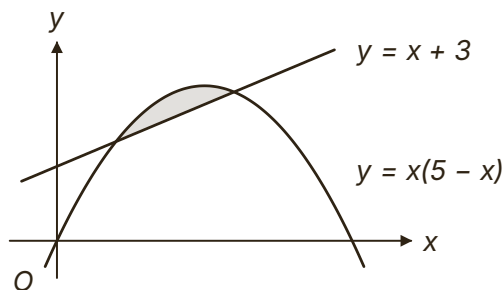
- B2** The sketch shows the [4 marks]
curve
 $y = x(x + 2)(x - 4)$.
Find the total area of
the two shaded regions.



Answer

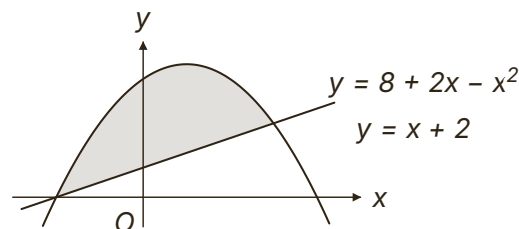
EXAMPLE 3

The sketch shows the curve $y = x(5 - x)$ and the line $y = x + 3$. Find the area of the shaded region between them.



Hint Find where they meet. Between those points the curve is on top, so integrate (curve minus line).

B3 The sketch shows the curve $y = 8 + 2x - x^2$ and the line $y = x + 2$. Find the area of the shaded region between them. [4 marks]



Answer

c Practice

A-Level

C1 Find the area of the finite region enclosed by the curves $y = x^2 + 1$ and $y = 9 - x^2$. [4 marks]

Answer

- C2** The finite region R is bounded by the curve $y = x^3 + 2$, the line $y = 29$ and the y -axis. Find the area of R . [3 marks]

Answer

- C3** Find the value of $\int_0^3 (x^2 - 4) dx$. Find also the total area of the regions bounded by $y = x^2 - 4$, the x -axis and the lines $x = 0$ and $x = 3$. [4 marks]

Answer

- D1** Priya writes: the area of the region between $y = x^3 - 4x$ and the x -axis is $\int_{-2}^2 (x^3 - 4x) dx = \left[\frac{x^4}{4} - 2x^2 \right]_{-2}^2 = 0$ [5 marks]

Priya is wrong. Explain her mistake, and find the correct area. Hence find the exact value of a , where $a > 2$, for which $\int_0^a (x^3 - 4x) dx = 0$.

Answer

- D2** $f(x) = -x^3 + 3x^2 + 6x - 8$. The sketch shows part of the curve $y = f(x)$.

- (a) Show that $(x - 1)$ [4 marks]

is a factor of $f(x)$.

Hence factorise

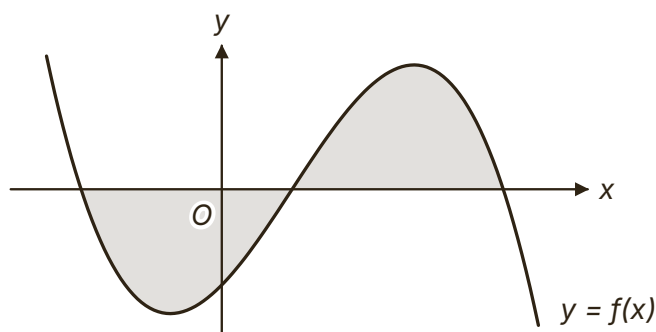
$f(x)$ completely

and write down the

x -coordinates

where the curve

meets the x -axis.



Answer

(b) Hence find the total area of the two shaded regions.

[6 marks]

Answer

Total for D2: 10 marks

- D3** A particle moves along a straight line. Its velocity, v m/s, t seconds after it starts, is

$$v = t^2 - 6t + 8, \quad 0 \leq t \leq 5$$

The displacement is the integral of v . The distance travelled is the total area between the graph of v and the t -axis.

- (a) Find the two times when the particle is at rest, and its displacement at $t = 5$ from where it started.

[5 marks]

Answer

(b) Hence find the total distance it travels in the 5 seconds.

[5 marks]

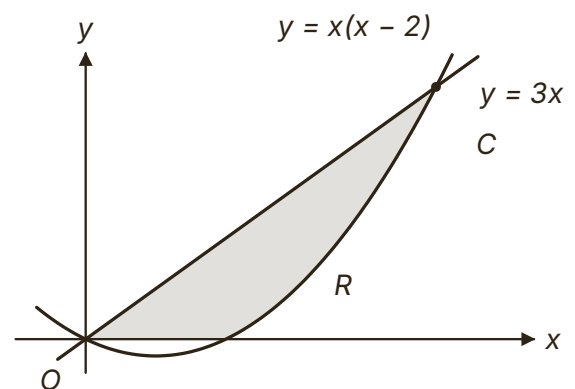
Answer

Total for D3: 10 marks

D4 The sketch shows the curve $y = x(x - 2)$ and the line $y = 3x$. The line meets the curve at O and at C . The region R , shaded, is bounded by the line OC , the x -axis and the curve.

(a) Find the coordinates of C . [2 marks]

Answer



(b) Hence find the area of R . [5 marks]

Answer

- (c) Find the area of the finite region enclosed by the line OC and the curve. [3 marks]

Answer

Total for D4: 10 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The finite region bounded by the curve $y = x(6 - x)$ and the x -axis [6 marks] is cut into two parts by the line $y = mx$, where $0 < m < 6$. The two parts have equal area. Find the exact value of m .

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Find the whole area first. Then find the area between the curve and the line in terms of m .
- Hint 2** The line meets the curve where $x(6 - x) = mx$, at $x = 0$ and $x = 6 - m$. The whole area is 36.
- Hint 3** $\int_0^{6-m} ((6 - m)x - x^2) dx = \frac{(6 - m)^3}{6}$. Set this equal to 18, so $(6 - m)^3 = 108$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find an area below the x -axis, and an area that crosses the axis. ☐ ☐ ☐

I can find the area between a curve and a line, or between two curves. ☐ ☐ ☐

I can combine an integral with a triangle to find an area. ☐ ☐ ☐