# Areas of sectors and segments

115 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-areas-of-sectors-and-segments/

Date

## A Do Now

Quick questions to start.

**A1** Needed today [2 marks]

A triangle has sides $8$ cm and $5$ cm with an angle of $\frac{\pi}{6}$ between them. Find its area.

Answer .....

**A2** Needed today [2 marks]

A sector has radius $6$ cm and angle $60°$. Find its exact area, using a fraction of the area of the circle.

Answer .....

**A3** Needed today [2 marks]

Find the exact value of $\sin\frac{2\pi}{3} \times \cos\frac{3\pi}{4}$.

Answer .....

**A4** Common slip [2 marks]

An arc of length $9$ cm is part of a circle of radius $6$ cm. Find the angle it subtends at the centre, in radians.

Answer .....



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## B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

**EXAMPLE 1**

A sector of a circle of radius $5$ cm has area $15$ cm². Find the angle of the sector, in radians, and its perimeter.

**Hint** Use $A = \frac{1}{2}r^2\theta$ with $\theta$ in radians. The perimeter is the arc and two radii.

**B1** A sector has angle $0.75$ radians and area $24$ cm². Find its radius and its perimeter. [4 marks]

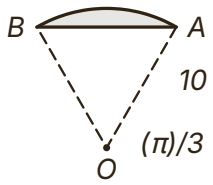
Answer .....



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**EXAMPLE 2**

The chord $AB$ of a circle with centre $O$ and radius $10$ cm subtends an angle of $\frac{\pi}{3}$ at $O$. Find the exact area of the minor segment cut off by $AB$.




**Hint** Segment $=$ sector $-$ triangle $OAB$. The triangle has area $\frac{1}{2}r^2 \sin\theta$.

**B2** The chord $PQ$ of a circle with centre $O$ and radius $8$ cm subtends an angle of $\frac{2\pi}{3}$ at $O$. Find the area of the minor segment, exactly and to 3 significant figures. [4 marks]

Answer .....


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**EXAMPLE 3**

A segment of a circle has angle $1.2$ radians at the centre and area $5.6$ cm². Find the radius of the circle, to 3 significant figures.

**Hint** Write the segment area as $\frac{1}{2}r^2(\theta - \sin\theta)$ and solve for $r$. Use radian mode.

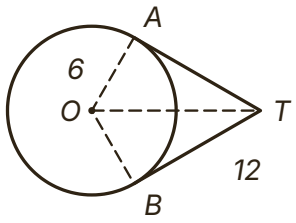
**B3** A segment of a circle has angle $2$ radians at the centre and area $30$ cm². Find the radius, to 3 significant figures. [4 marks]

Answer .....


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**EXAMPLE 4**

$TA$ and $TB$ are tangents from the point $T$ to a circle with centre $O$ and radius $6$ cm. $OT = 12$ cm. Find the exact area of the region bounded by $TA$, $TB$ and the minor arc $AB$.




**Hint** A tangent is perpendicular to the radius. Find angle $AOT$ from the right-angled triangle $OAT$, then take the sector away from the kite $OATB$.

**B4** $TA$ and $TB$ are tangents from $T$ to a circle with centre $O$ and radius $5$ cm. $OT = 5\sqrt{2}$ cm. Find the area of the region bounded by $TA$, $TB$ and the minor arc $AB$, exactly and to 3 significant figures. [5 marks]

Answer .....


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**EXAMPLE 5**

A sector has perimeter $20$ cm. Find the greatest possible area of the sector and the angle that gives it.

**Hint** Write $\theta$ in terms of $r$ from the perimeter, then write the area in terms of $r$ alone and complete the square or differentiate.

**B5** A sector has perimeter $36$ cm. Find the greatest possible area of the sector and the angle that gives it. [5 marks]

Answer .....


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**C1** Kai finds the area of the minor segment of a circle of radius $9$ cm cut off by a chord that subtends $60^\circ$ at the centre. He writes: "area $= \frac{1}{2} \times 81 \times (60 - \sin 60) = 2395$". Find his mistake and give the correct area, exactly and to 3 significant figures. [5 marks]

Answer .....


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**C2** A segment has radius $10$ cm and angle $1.5$ radians at the centre. Mia's calculator gives the area as $73.7$ cm². Explain what she has done wrong and find the correct area, to 3 significant figures. [4 marks]

Answer .....

**C3** A chord of a circle of radius $6$ cm subtends an angle of $1$ radian at the centre. Find the area of the major segment, exactly and to 3 significant figures. [5 marks]

Answer .....


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**C4** A shape is the part of a sector of radius $7$ cm and angle $1.2$ radians that lies outside the sector of radius $4$ cm with the same centre and angle. Find its area and its perimeter. [5 marks]

Answer .....


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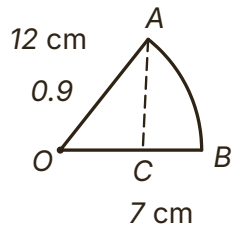
**C5** The minor segment cut off by a chord $AB$ of a circle with centre $O$ has the same area as triangle $OAB$. Angle $AOB = \theta$ radians. Show that $\theta = 2\sin\theta$, and show that the root lies between $1.8$ and $2$. [5 marks]

## D Exam-style questions

Edexcel exam style

**D1** The diagram shows a sector $OAB$ of a circle with centre $O$, radius $12$ cm and angle $0.9$ radians. The point $C$ lies on $OB$ with $OC = 7$ cm. The region $R$ is bounded by the arc $AB$ and the straight lines $BC$ and $CA$.

**(a)** Find the area of triangle $OAC$, to 3 significant figures. [2 marks]




Answer .....



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**(b)** Find the length $AC$, to 3 significant figures. [3 marks]

Answer .....

**(c)** Find the perimeter of $R$, to 3 significant figures. [3 marks]

Answer .....

**(d)** Find the area of $R$, to 3 significant figures. [2 marks]

Answer .....

**Total for D1: 10 marks**


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**D2**

A circle with centre $O$ has radius $9$ cm. The chord $AB$ has length $14$ cm. Ravi works out angle $AOB = 1.782$ radians and writes: "area of the major segment $=$ major sector $-$ triangle $OAB = \frac{1}{2} \times 81 \times (2\pi - 1.782) - \frac{1}{2} \times 81 \times \sin 1.782 = 143$ cm²".

Ravi is wrong.

**(a)** Explain the mistake in Ravi's method. [2 marks]

**(b)** Show that angle $AOB = 1.782$ radians, to 3 decimal places. [2 marks]


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**(c)** Find the correct area of the major segment, to 3 significant figures. [3 marks]

Answer .....

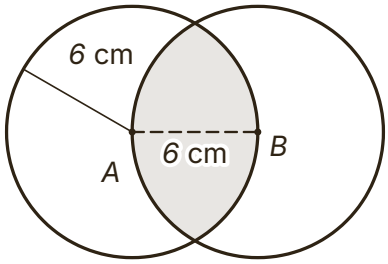
**(d)** Find the perimeter of the major segment, to 3 significant figures. [2 marks]

Answer .....

**Total for D2: 9 marks**


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**D3** Two circles each have radius $6$ cm, and the centre of each lies on the other circle. Show that the area of the region inside both circles is $24\pi - 18\sqrt{3}$ cm². [8 marks]




**D4** A sector of a circle has radius $r$ cm and area $50$ cm².

**(a)** Show that the perimeter $P$ cm of the sector is given by $P = 2r + \dfrac{100}{r}$. [2 marks]


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**(b)** Find the exact value of $r$ for which $P$ is least. [3 marks]

Answer .....

**(c)** Find the least perimeter, exactly, and the angle of the sector that gives it. [3 marks]

Answer .....

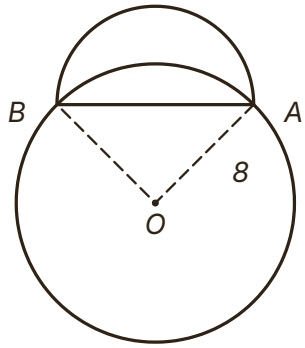
**(d)** Show that this value of $r$ gives a least value, not a greatest. [2 marks]

**Total for D4: 10 marks**


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**D5** A circle with centre $O$ has radius $8$ cm. The chord $AB$ has length $8\sqrt{2}$ cm. A semicircle is drawn on $AB$ as diameter, outside the circle. The crescent is the part of the semicircle that lies outside the circle.

**(a)** Show that angle $AOB = \dfrac{\pi}{2}$. [2 marks]




**(b)** Find the exact area of the minor segment cut off by $AB$. [3 marks]

Answer .....


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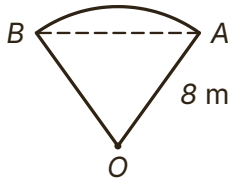
**(c)** Show that the area of the crescent is $32$ cm², the same as the area of triangle $OAB$. [3 marks]

**Total for D5: 8 marks**

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**D6** A garden sprinkler at $O$ waters a sector of a lawn with radius $8$ m. The watered area is $40$ m². A straight path joins the ends $A$ and $B$ of the arc.

**(a)** Find the angle of the sector, in radians. [2 marks]




Answer .....


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**(b)** Find the length of the path $AB$, to 3 significant figures. [2 marks]

Answer .....

**(c)** Find the watered area on the far side of the path from $O$, to 3 significant figures. [3 marks]

Answer .....

**Total for D6: 7 marks**


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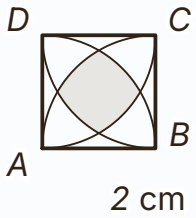
## E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

**E1** [9 marks]

$ABCD$ is a square of side $2$ cm. Four arcs of radius $2$ cm are drawn inside the square, each with its centre at a corner and joining the two corners next to it. Show that the area of the shaded region, which lies inside all four arcs, is $4 + \frac{4\pi}{3} - 4\sqrt{3}$ cm².





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## Extension hints

Cover these with paper. Uncover one at a time.

**E1**

**Hint 1** Put the square on axes with $A$ at the origin. Where do the arcs centred at $A$ and $B$ cross? The crossing point is the same distance from both.

**Hint 2** The four corners of the shaded region make a square. Find the length of its diagonal.

**Hint 3** Each curved edge of the shaded region is an arc of one of the circles. Find the angle it makes at its centre, then add four segments to the inner square.

## How did it go?

Colour one circle on each line: not yet, nearly, yes.

| | |
|---|---|
| I can find the area of a sector with $A = \frac{1}{2}r^2\theta$, with $\theta$ in radians. | ○ ○ ○ |
| I can find the area of a segment as a sector minus a triangle, exactly or to a given accuracy. | ○ ○ ○ |
| I can work backwards from an area to a radius or an angle. | ○ ○ ○ |
| I can split a harder shape into sectors, triangles and segments, and find its greatest or least area or perimeter. | ○ ○ ○ |


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