

Areas of triangles and solving triangle problems

72 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-areas-of-triangles-and-triangle-problems/

Date _____

A Do Now

Quick questions to start.

- A1** [Last lesson] [2 marks] A triangle has sides 5 cm, 7 cm and 8 cm. Find the angle opposite the 7 cm side.
- A2** [Needed today] [2 marks] Solve $\sin x = 0.35$ for $0^\circ < x < 180^\circ$, to 1 d.p.

Answer $x = \dots\dots$

Answer

- A3** [From chapter 8] [2 marks] Find the coefficient of x^3 in the expansion of $(2 + x)^6$.
- A4** [From chapter 2] [2 marks] Write $3 + 4x - x^2$ in the form $p - (x - q)^2$.

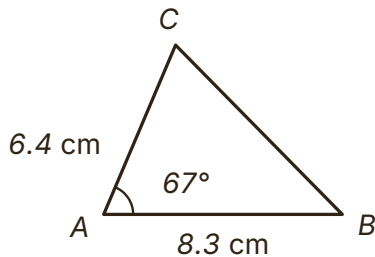
Answer

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

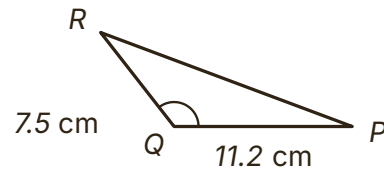
Find the area of triangle ABC , to 3 s.f.



Hint Area = $\frac{1}{2}ab \sin C$, where C is the angle between the two sides you know.

B1 Angle $PQR = 128^\circ$. [2 marks]

Find the area of triangle PQR , to 3 s.f.

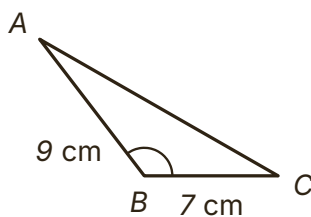


Not drawn accurately

Answer

EXAMPLE 2

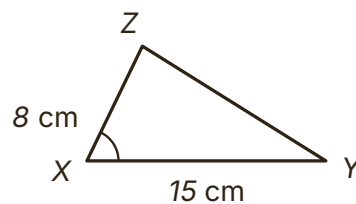
Triangle ABC has area 25 cm^2 and angle ABC is obtuse. Find angle ABC and AC .



Hint Put the area into $\frac{1}{2}ac \sin B$ and solve for $\sin B$. Of the two angles, take the obtuse one. Then use the cosine rule.

B2 Triangle XYZ has area [4 marks]

54 cm^2 and angle YXZ is acute. Find angle YXZ and YZ .

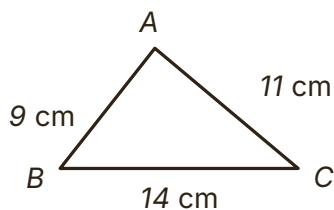


Not drawn accurately

Answer angle = ... YZ = ...

EXAMPLE 3

Find the area of this triangle, to 3 s.f.

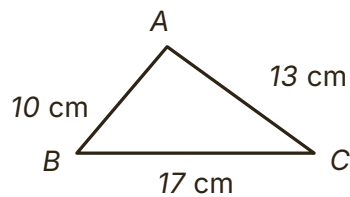


Hint You need an angle first. Find any angle with the cosine rule, then use $\frac{1}{2}ab \sin C$ with the two sides beside it.

B3

Find the area of this triangle, to 3 s.f.

[4 marks]

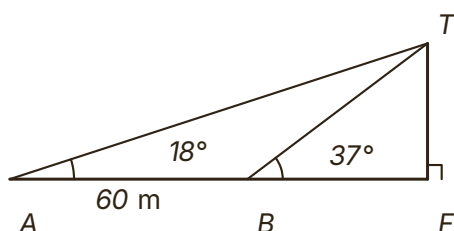


Not drawn accurately

Answer

EXAMPLE 4

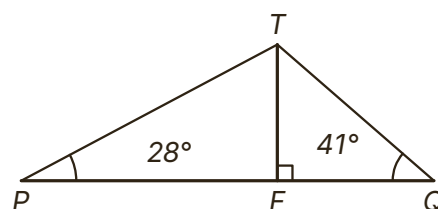
A , B and the foot F of a tower FT are in a line on level ground. Find the height FT .



Hint In triangle ABT the angle at T is $37^\circ - 18^\circ$. Use the sine rule for BT , then right-angled trigonometry in triangle BFT .

B4

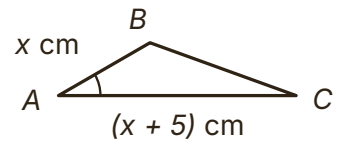
P , F and Q are in a line [4 marks]
on level ground and
 $PQ = 85$ m. Find the
height of the tower FT .



Not drawn accurately

Answer

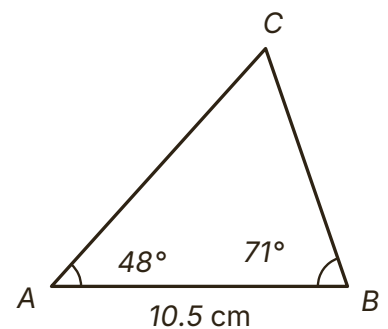
- C1** In triangle ABC , $AB = x$ cm, $AC = (x + 5)$ cm and angle $BAC = 30^\circ$. The area of the triangle is 9 cm^2 . Find the value of x . [4 marks]



Not drawn accurately

Answer $x = \dots\dots$

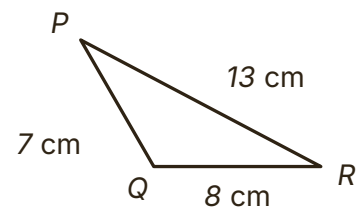
- C2** In triangle ABC , $AB = 10.5$ cm, angle $CAB = 48^\circ$ and angle $ABC = 71^\circ$. Find the area of the triangle, to 3 s.f. [4 marks]



Not drawn accurately

Answer

- C3** In triangle PQR , $PQ = 7$ cm, $QR = 8$ cm and $PR = 13$ cm. Show that angle $PQR = 120^\circ$, and find the exact area of the triangle. [4 marks]

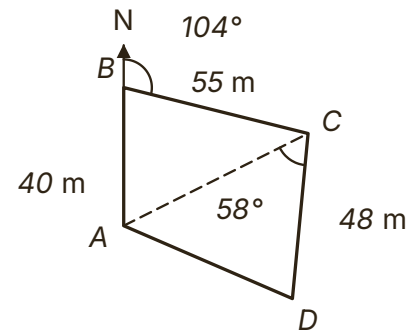


Not drawn accurately

Answer area = ...

- D1** A garden $ABCD$ has four straight fences. B is 40 m due north of A . C is 55 m from B on a bearing of 104° . $CD = 48$ m and angle $ACD = 58^\circ$.

- (a) Find the length AC , to 3 s.f. [3 marks]



Not drawn accurately

Answer

- (b) Hence find the area of the garden, to 3 s.f. [4 marks]

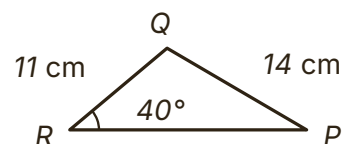
Answer

Total for D1: 7 marks

- D2** In triangle PQR , $PQ = 14$ cm, $QR = 11$ cm and angle $QRP = 40^\circ$. Sam writes: area = $\frac{1}{2} \times 14 \times 11 \times \sin 40^\circ = 49.5 \text{ cm}^2$.

Sam's answer is wrong.

- (a) Explain the mistake Sam has made. [2 marks]



Not drawn accurately

- (b) Find angle QPR . Explain why it has only one possible value. [3 marks]

Answer angle $QPR = \dots\dots$

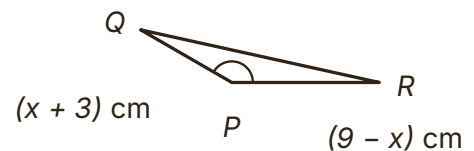
- (c) Hence find the correct area of the triangle, to 3 s.f. [3 marks]

Answer

Total for D2: 8 marks

- D3** In triangle PQR , $PQ = (x + 3)$ cm, $PR = (9 - x)$ cm and angle $QPR = 150^\circ$, where $-3 < x < 9$. The area of the triangle is A cm².

- (a) Show that [3 marks]
 $A = \frac{1}{4}(27 + 6x - x^2)$.



Not drawn accurately

- (b) Hence find the maximum value of A , and the value of x at which it happens. [3 marks]

Answer $A = \dots\dots$ $x = \dots\dots$

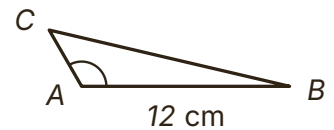
- (c) Given that $A = 8$, find the two possible values of x . [3 marks]

Answer $x = \dots\dots$ or $\dots\dots$

Total for D3: 9 marks

D4 Triangle ABC has perimeter 30 cm. $AB = 12$ cm and angle $BAC = 120^\circ$.

- (a) Show that $AC = 3.75$ cm. [4 marks]



Not drawn accurately

- (b) Hence find the exact area of the triangle. [2 marks]

Answer

- (c) Find the shortest distance from A to the line BC , to 3 s.f. [2 marks]

Answer

Total for D4: 8 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** A triangle has sides 10 cm, 17 cm and 21 cm. A circle is drawn inside the triangle, touching all three sides. Find the exact radius of the circle. [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Join the centre of the circle to the three corners. This splits the triangle into three triangles, each with height equal to the radius.
- Hint 2** Area = $\frac{1}{2}r(10 + 17 + 21) = 24r$. Now find the area another way: the cosine rule for one angle, then $\frac{1}{2}ab \sin C$.
- Hint 3** $\cos C = \frac{10^2 + 17^2 - 21^2}{2 \times 10 \times 17} = -\frac{13}{85}$, so
 $\sin C = \frac{84}{85}$ and the area is
 $\frac{1}{2} \times 10 \times 17 \times \frac{84}{85} = 84$. So $24r = 84$ and
 $r = 3.5$

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the area of a triangle using half $ab \sin C$. ○ ○ ○

I can find an angle or a side when I know the area. ○ ○ ○

I can choose the sine rule, the cosine rule or the area formula. ○ ○ ○

I can solve problems with heights, bearings and quadrilaterals. ○ ○ ○