Arithmetic sequences and series

113 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-arithmetic-sequences-and-series/

Date

A Do Now

Quick questions to start. They use what you need today.

A1 Needed today [2 marks]

Solve $n^2 + 3n - 130 = 0$, where n is a positive integer.

Answer .....

A2 Needed today [2 marks]

Find an expression for the nth term of $7, 11, 15, 19, \ldots$

Answer .....

A3 Common slip [2 marks]

An arithmetic sequence has first term 5 and common difference 3. Find the 20th term.

Answer .....

A4 Needed today [3 marks]

Find the smallest positive integer n for which $n^2 - 9n - 52 > 0$.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

The arithmetic sequence $2, 9, 16, 23, \ldots$ has nth term u_n. Find u_{30}, and find which term is equal to 359.

Hint Use $u_n = a + (n-1)d$. For the position, set $u_n = 359$ and solve for n.

B1 [4 marks]

The arithmetic sequence $85, 81, 77, \ldots$ has nth term u_n. (a) Find u_{25}. (b) Find which term is equal to -51.

Answer .....

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EXAMPLE 2

In an arithmetic series $u_5 = 23$ and $u_{12} = 51$. Find the first term, the common difference and S_{20}.

Hint Write two equations in a and d and subtract them.

B2 In an arithmetic series $u_4 = 17$ and $u_{15} = -27$. Find the first term, the common difference and S_{25}. [5 marks]

Answer .....

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EXAMPLE 3

Prove that the sum of the first n terms of an arithmetic series with first term a and common difference d is $S_n = \frac{n}{2}(2a + (n-1)d)$.

Hint Write the series forwards, then backwards underneath, and add the two lines term by term.

B3 Prove that $1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$. [4 marks]

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EXAMPLE 4

For the series $3 + 7 + 11 + \cdots$, find the least number of terms for which the sum is greater than 500.

Hint Write S_n as a quadratic in n, solve $S_n = 500$, then round to the next whole number. Check the two values either side.

B4 For the series $50 + 46 + 42 + \cdots$, find the least number of terms for which the sum is negative. [5 marks]

Answer .....

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EXAMPLE 5

Find the sum of all the multiples of 7 between 100 and 500.

Hint Find the first and last multiples, then the number of terms, before you use a sum formula.

B5 Find the sum of all the multiples of 9 between 200 and 800. [5 marks]

Answer .....

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C1 A student finds the sum of the first 15 terms of $8 + 5 + 2 + \cdots$ and writes $S_{15} = \dfrac{15}{2}(2 \times 8 + 15 \times (-3)) = -217.5$. Explain the mistake and find the correct value of S_{15}. [4 marks]

Answer $S_{15} = \ldots\ldots$

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C2 Find the sum of the integers from 1 to 100 that are not multiples of 4. [5 marks]

Answer .....

C3 The sum of the first n terms of a series is $S_n = 3n^2 + 2n$. Find an expression for u_n, show that the series is arithmetic, and find u_{10}. [5 marks]

Answer .....

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C4 The sum of the first n terms of $4 + 7 + 10 + \cdots$ is 1079. Find n. [4 marks]

Answer .....

C5 Find the sum of the series $7 + 11 + 15 + \cdots + 403$. [4 marks]

Answer .....

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D1 The first three terms of an arithmetic sequence are $(k+4)$, $(3k-1)$ and $(4k+5)$, where k is a constant.

(a) Find the value of k. [2 marks]

Answer .....

(b) Find the 20th term of the sequence. [2 marks]

Answer .....

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(c) Find the least value of n for which the sum of the first n terms is greater than 10000. [5 marks]

Answer .....

Total for D1: 9 marks

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D2

> Raj wants the greatest n for which the sum of the first n terms of $120 + 113 + 106 + \cdots$ is positive. He writes "$S_n = \frac{n}{2}(240 - 7n)$, so $S_n > 0$ when $n < \frac{240}{7} = 34.3$. The greatest value is $n = 34$."

Raj is wrong.

(a) Find Raj's mistake and give the correct greatest n. [4 marks]

(b) Find the greatest value of S_n, and the value of n at which it occurs. [4 marks]

Answer .....

Total for D2: 8 marks

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D3 The sum of the first 10 terms of an arithmetic series is 175. The sum of the next 10 terms is 575. Find the first term and the common difference. [7 marks]

Answer .....

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D4 The first three terms of a sequence are $\log_2 3$, $\log_2 12$ and $\log_2 48$.

(a) Show that the sequence is arithmetic and state the common difference. [3 marks]

(b) Show that $S_{10} = 10\log_2 3 + 90$. [3 marks]

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(c) Find the least value of n for which $S_n > 200$. [4 marks]

Answer .....

Total for D4: 10 marks

D5 Sequence A is $5, 12, 19, \ldots$ and sequence B is $203, 199, 195, \ldots$. Both are arithmetic.

(a) Show that the 19th terms of A and B are equal. [3 marks]

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(b) Find the least value of n for which the sum of the first n terms of A is greater than the sum of the first n terms of B. [6 marks]

Answer .....

Total for D5: 9 marks

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D6 Show that the sum of all the three-digit numbers that leave a remainder of 2 when divided by 5 is 98910. [7 marks]

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 An arithmetic series has first term 1 and common difference d, where d is a positive integer. For some $n \geq 2$, the sum of the first n terms is 1000. Find every possible value of d. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write $S_n = 1000$ and clear the fraction: $n(2 + (n-1)d) = 2000$.

Hint 2 So n must be a factor of 2000, and $\dfrac{2000}{n} - 2$ must divide exactly by $n - 1$.

Hint 3 Since $d \geq 1$, $n(n+1) \leq 2000$, so $n \leq 44$. Check each factor of 2000 up to 44.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the nth term of an arithmetic sequence and which term has a given value.	○ ○ ○
I can find the first term and common difference from two facts.	○ ○ ○
I can prove the formula for the sum of an arithmetic series.	○ ○ ○
I can find the least or greatest n for a condition on S_n.	○ ○ ○

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