



Binomial estimation

70 marks

Date _____

A Do Now

Quick questions to start.

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|--|---|
| <p>A1 [Last lesson] [2 marks]</p> <p>Find the coefficient of x^2 in $(1 - 4x)^7$.</p> | <p>A2 [Last lesson] [2 marks]</p> <p>Find the constant term in $(x - \frac{3}{x})^4$.</p> |
|--|---|

Answer

Answer

- | | |
|--|--|
| <p>A3 [Needed today] [2 marks]</p> <p>Find x when $1 - 3x = 0.988$.</p> | <p>A4 [Needed today] [2 marks]</p> <p>Find the percentage error when 12.4 is used for 12.5.</p> |
|--|--|

Answer

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find the first four terms of $(1 - \frac{1}{4}x)^8$.
Use them to estimate 0.97^8 to 4 d.p.

Hint Choose x so that $1 - \frac{1}{4}x = 0.97$.

B1 Find the first four terms [4 marks]
of $(1 + \frac{1}{5}x)^6$. Use them
to estimate 1.04^6 to 4
d.p.

Answer

EXAMPLE 2

Find the first three terms of $(2 - x)^7$.
Use them to estimate 1.99^7 to 2 d.p.

Hint $2 - x = 1.99$ gives $x = 0.01$.

B2 Find the first three [3 marks]
terms of $(3 + \frac{1}{2}x)^6$. Use
them to estimate 3.01^6
to 2 d.p.

Answer

EXAMPLE 3

x is small, so terms in x^3 and higher can be ignored. Show that $(1 + 2x)(1 - x)^6 \approx 1 - 4x + 3x^2$.

Hint Expand $(1 - x)^6$ only up to x^2 , then multiply out.

B3 x is small, so terms in x^3 and higher can be ignored. [3 marks]

$(3 - x)(1 + 2x)^5 \approx a + bx + cx^2$. Find a , b and c .

Answer

EXAMPLE 4

Use the first three terms of $(1 - 2x)^9$ to estimate 0.996^9 . Find the percentage error, to 2 s.f.

Hint Percentage error

$$= \frac{\text{estimate} - \text{true}}{\text{true}} \times 100.$$

B4 Use the first three terms of $(1 + 3x)^8$ to estimate 1.003^8 . Find the percentage error, to 2 s.f. [4 marks]

Answer

- C1** x is small, so terms in x^3 and higher can be ignored. [4 marks]
 $(1 - x)(1 + kx)^6 \approx 1 + 11x + ax^2$. Find the constants k and a .

Answer

- C2** Without a calculator, use the expansion of $(1 + x)^5$ to find the exact [3 marks]
value of 1.1^5 .

Answer

- C3** The first three terms of $(1 + \frac{1}{2}x)^{10}$ are used to estimate 1.05^{10} . Work [4 marks]
out the estimate. Is it too large or too small? Give a reason.

- D1** Priya puts $x = 0.3$ into the first three terms of $(1 + x)^8$ and gets $1.3^8 \approx 5.92$. Her calculator gives 8.157. She says: "Binomial estimates do not work."

Priya's conclusion is wrong.

- (a) Find the percentage error in Priya's estimate, to 2 significant figures. [2 marks]

Answer

- (b) Explain why her estimate is poor. [2 marks]

- (c) Hence use the same three terms, with a suitable value of x , to estimate 1.03^8 . Find the percentage error in this estimate. [3 marks]

Answer

Total for D1: 7 marks

D2 The probability that a box of 50 bulbs holds no faulty bulb is modelled as $(1 - p)^{50}$, where p is the probability that one bulb is faulty.

(a) Find the first three terms, in ascending powers of p , of $(1 - p)^{50}$. [2 marks]

Answer

(b) Explain why your answer to (a) is only a good approximation when p is small. [1 mark]

(c) The factory wants the probability of no faulty bulb in a box to be at least 0.9. Hence estimate the largest value of p , to 3 s.f. [4 marks]

Answer

Total for D2: 7 marks

D3 In this question x is small, so terms in x^3 and higher can be ignored.

(a) Find the first three terms, in ascending powers of x , of $(2 - x)^6$. [2 marks]

Answer

(b) Hence show that $(1 + 3x)(2 - x)^6 \approx 64 - 336x^2$. [3 marks]

(c) Hence estimate 1.03×1.99^6 . State the value of x you use and give your answer to 4 d.p. [3 marks]

Answer

Total for D3: 8 marks

D4 In this question, do not use a calculator.

(a) Expand $(3 + 2x)^5$, giving each coefficient as an integer. [4 marks]

Answer

(b) Hence find the exact value of $3.02^5 + 2.98^5$. [3 marks]

Answer

- (c) Use the first three terms to estimate 3.02^5 to 2 d.p. Explain why the other terms do not change this. [3 marks]

Total for D4: 10 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Without a calculator, show that $1.01^{100} > 2.7$. [5 marks]

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Write 1.01^{100} as $(1 + 0.01)^{100}$.
Hint 2 Every term is positive, so a sum of the first terms is too small.
Hint 3 You need the terms up to 0.01^5 .

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can estimate a power from the first terms of an expansion. ☐ ☐ ☒

I can approximate a product, ignoring x cubed and higher. ☐ ☐ ☒

I can find a percentage error and say why x must be small. ☐ ☐ ☒

I can say if an estimate is too large or too small. ☐ ☐ ☒