Expanding $(1 + x)^n$ for any n

117 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-binomial-expansion-of-1-plus-x-to-any-power/

Date

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A Do Now

Quick questions to start.

A1 Needed today [3 marks]

Expand $(1+2x)^4$ fully.

Answer .....

A2 Needed today [2 marks]

Work out $\dfrac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}$.

Answer .....

A3 Common slip [2 marks]

Simplify $(-3x)^2$ and $(-3x)^3$.

Answer .....

A4 Needed today [2 marks]

Write $\dfrac{1}{\sqrt{1+x}}$ and $\dfrac{1}{(1-x)^2}$ as powers of a bracket.

Answer .....

A5 Common slip [2 marks]

Solve $|3x| < 1$.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find the first four terms of the binomial expansion of $(1-x)^{-3}$, in ascending powers of x. State the values of x for which the expansion is valid.

Hint Use $(1+y)^n = 1 + ny + \frac{n(n-1)}{2!}y^2 + \frac{n(n-1)(n-2)}{3!}y^3 + \ldots$ with $n = -3$ and $y = -x$.

B1 [4 marks]

Find the first four terms of the binomial expansion of $(1+x)^{-4}$, in ascending powers of x. State the values of x for which it is valid.

Answer .....

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EXAMPLE 2

Find the first four terms of the expansion of $\sqrt{1-4x}$ and state the values of x for which it is valid.

Hint Write it as $(1-4x)^{\frac{1}{2}}$. Keep $(-4x)$ in a bracket every time you raise it to a power.

B2 Find the first four terms of the expansion of $(1+3x)^{-\frac{1}{3}}$ and state the values of x for which it is valid. [5 marks]

Answer .....

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EXAMPLE 3

Use your expansion of $\sqrt{1-4x}$ from the example above, with $x = 0.01$, to find an approximation to $\sqrt{6}$, to 5 decimal places.

Hint Work out $\sqrt{1-4(0.01)} = \sqrt{0.96}$ and write it in terms of $\sqrt{6}$.

B3 Use your expansion of $(1+3x)^{-\frac{1}{3}}$ from B2 with $x = -\dfrac{1}{27}$ to find an approximation to $\sqrt[3]{9}$, to 3 decimal places. [3 marks]

Answer .....

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EXAMPLE 4

The first three terms of the expansion of $(1+kx)^n$ are $1-6x+27x^2$. Find k and n.

Hint Write $nk=-6$ and $\frac{n(n-1)}{2}k^2=27$. Use the first equation to remove k from the second.

B4 [6 marks]

The first three terms of the expansion of $(1+kx)^n$ are $1-4x-8x^2$. Find k and n, the term in x^3, and the values of x for which the expansion is valid.

Answer .....

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EXAMPLE 5

Find the expansion of $(2 - x)\sqrt{1 + 4x}$ in ascending powers of x, up to and including the term in x^2.

Hint Expand $\sqrt{1 + 4x}$ to the x^2 term, then multiply by $(2 - x)$ and collect only up to x^2.

B5 Find the expansion of $\dfrac{3 + x}{\sqrt{1 - 2x}}$ in ascending powers of x, up to and including the term in x^2. [4 marks]

Answer .....

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C1 Mia expands $(1-2x)^{-\frac{1}{2}}$. She writes: "$1+\left(-\frac{1}{2}\right)(-2x)+\frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2!}(-2x^2)=1+x-\frac{3}{4}x^2$." Find her mistake and give the correct first four terms. [4 marks]

Answer .....

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C2 Find the first three terms of the expansion of $(1+5x)^{-1}$ and state the values of x for which it is valid. Ben substitutes $x = 0.5$ into the series to find $\frac{1}{3.5}$. Explain why this does not work. [4 marks]

Answer .....

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C3 (a) Find the first three terms of the expansion of $(1-8x)^{\frac{1}{2}}$. (b) By substituting $x = 0.01$, find an approximation to $\sqrt{23}$, to 3 decimal places. (c) Explain why $x = 0.25$ cannot be used in the expansion. [6 marks]

Answer .....

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C4 In the expansion of $(1 + ax)^{-3}$, where a is a non-zero constant, the coefficient of x^3 is equal to the coefficient of x^2. Find a and state the values of x for which the expansion is valid. [5 marks]

Answer .....

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D1 $f(x) = (1 - 9x)^{-\frac{1}{3}}$

(a) Find the first four terms of the binomial expansion of $f(x)$, in ascending powers of x. [4 marks]

Answer .....

(b) Show that $f\left(-\frac{2}{225}\right) = \frac{\sqrt[3]{25}}{3}$, and explain why $x = -\frac{2}{225}$ is a suitable value to use in the expansion. [3 marks]

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(c) Hence find an approximation to $\sqrt[3]{25}$, giving your answer to 4 decimal places. [3 marks]

Answer .....

Total for D1: 10 marks

D2

Leo writes: "$(1+2x)^{-3} = 1 - 6x + 24x^2 - 80x^3 + \ldots$, and this is true for all values of x."

Leo's terms are correct, but his statement is wrong.

(a) Explain why Leo's statement is wrong, and state the correct values of x. [2 marks]

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(b) Given that $(a + bx)(1 + 2x)^{-3} = 2 + cx^2 + \ldots$, where the expansion has no term in x, find the values of the constants a, b and c. [6 marks]

Answer .....

Total for D2: 8 marks

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D3 [9 marks]

Find the binomial expansion of $\sqrt{\dfrac{1+2x}{1-2x}}$, in ascending powers of x, up to and including the term in x^3. By substituting $x = \dfrac{1}{20}$, find an approximation to $\sqrt{11}$, giving your answer to 3 decimal places.

Answer .....

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D4 In the binomial expansion of $(1 + kx)^n$, where k and n are constants and n is not a positive integer, the coefficients of x, x^2 and x^3 are in the ratio $2 : 9 : 45$.

(a) Find the value of n and the value of k. [5 marks]

Answer .....

(b) Find the coefficient of x and the coefficient of x^3. [2 marks]

Answer .....

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(c) State the values of x for which the expansion is valid, and explain why it cannot be used to find $(1+kx)^n$ when $x = \frac{1}{5}$. [2 marks]

Total for D4: 9 marks

D5 $g(x) = (1+4x)^{\frac{3}{2}}$

(a) Find the first four terms of the binomial expansion of $g(x)$, in ascending powers of x. [4 marks]

Answer .....

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(b) Use your expansion with $x = 0.05$ to find an estimate of $1.2^{\frac{3}{2}}$. Find the percentage error of this estimate, giving your answer to 2 significant figures. [4 marks]

Answer .....

(c) Explain why using $x = 0.3$ in the expansion would not give a reliable estimate of $2.2^{\frac{3}{2}}$. [2 marks]

Total for D5: 10 marks

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D6 $h(x) = \dfrac{2+x}{\sqrt{1+x}}, \quad |x| < 1$

(a) Show that, for small values of x, $h(x) \approx 2 + \dfrac{1}{4}x^2$. [4 marks]

(b) Use this approximation to estimate $h(0.1)$. [2 marks]

Answer .....

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(c) Find the term in x^3 in the expansion of $h(x)$. [3 marks]

Answer .....

Total for D6: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 [10 marks]

Show that the coefficient of x^r in the binomial expansion of $(1-4x)^{-\frac{1}{2}}$ is $\binom{2r}{r}$. Hence prove that, for every positive integer r, $\displaystyle\sum_{k=0}^{r}\binom{2k}{k}\binom{2r-2k}{r-k}=4^r$.

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write the general coefficient as $\frac{n(n-1)\cdots(n-r+1)}{r!}(-4)^r$ with $n=-\frac{1}{2}$, and take out a factor of $\left(-\frac{1}{2}\right)^r$.

Hint 2 The odd numbers $1\times3\times5\times\cdots\times(2r-1)$ equal $\frac{(2r)!}{2^r\,r!}$, because $(2r)!$ is the odd numbers times the even numbers $2\times4\times\cdots\times2r=2^r r!$.

Hint 3 Square the whole series: $\left((1-4x)^{-\frac{1}{2}}\right)^2=(1-4x)^{-1}$. Compare the coefficient of x^r on each side.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can write the first four terms of (1 + bx)ⁿ when n is negative or a fraction.	○ ○ ○
I can state the values of x for which the expansion is valid and explain why.	○ ○ ○
I can choose a value of x to approximate a root and say how accurate it is.	○ ○ ○
I can find unknown constants from given coefficients.	○ ○ ○

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