Expanding (a + bx)ⁿ

115 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-binomial-expansion-of-a-plus-bx/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Find the first three terms of the expansion of $(1+x)^{-\frac{1}{2}}$.

Answer .....

A2 Needed today [3 marks]

Work out $4^{-\frac{1}{2}}$, $8^{\frac{2}{3}}$ and $\left(\frac{1}{27}\right)^{-\frac{1}{3}}$.

Answer .....

A3 Common slip [2 marks]

Write $(9+3x)^{\frac{1}{2}}$ in the form $k(1+cx)^{\frac{1}{2}}$.

Answer .....

A4 Needed today [2 marks]

Solve $\left|\frac{2x}{5}\right| < 1$.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find the first four terms of the binomial expansion of $(4+x)^{-\frac{1}{2}}$, in ascending powers of x. State the values of x for which it is valid.

Hint Take out $4^{-\frac{1}{2}}$ first: $(4+x)^{-\frac{1}{2}} = 4^{-\frac{1}{2}}\left(1+\frac{x}{4}\right)^{-\frac{1}{2}}$.

B1 Find the first four terms of the binomial expansion of $(9-2x)^{\frac{1}{2}}$, in ascending powers of x. State the values of x for which it is valid. [5 marks]

Answer .....

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EXAMPLE 2

Find the first three terms of the binomial expansion of $(2+3x)^{-3}$, in ascending powers of x. State the values of x for which it is valid.

Hint $(2+3x)^{-3} = 2^{-3}\left(1+\frac{3x}{2}\right)^{-3}$. The factor is $2^{-3} = \frac{1}{8}$, not 2 and not 8.

B2 [5 marks]

Find the first four terms of the binomial expansion of $\frac{1}{(3-2x)^2}$, in ascending powers of x. State the values of x for which it is valid.

Answer .....

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EXAMPLE 3

Find the expansion of $\dfrac{3+x}{\sqrt{4-x}}$ in ascending powers of x, up to and including the term in x^2.

Hint Write it as $(3+x)(4-x)^{-\frac{1}{2}}$ and expand $(4-x)^{-\frac{1}{2}}$ to the x^2 term first.

B3 Find the expansion of $\dfrac{1+2x}{3+x}$ in ascending powers of x, up to and including the term in x^2. [5 marks]

Answer .....

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EXAMPLE 4

The expansion of $(a + 3x)^{-2}$, where a is a positive constant, begins $\frac{1}{4} + px + qx^2$. Find a, p and q.

Hint The constant term is a^{-2}. Set $a^{-2} = \frac{1}{4}$ first, then expand.

B4 [6 marks]

The expansion of $(a + bx)^{\frac{1}{3}}$ begins $2 + \frac{1}{4}x + \ldots$. Find a and b, the term in x^2, and the values of x for which the expansion is valid.

Answer .....

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EXAMPLE 5

Use the first three terms of the expansion of $(8-x)^{\frac{1}{3}}$ to find an approximation to $\sqrt[3]{7.9}$, to 5 decimal places.

Hint $7.9 = 8 - 0.1$, so use $x = 0.1$. Check it is inside the range of validity.

B5 Use the first three terms of the expansion of $(25+x)^{\frac{1}{2}}$ to find an approximation to $\sqrt{24}$, to 3 decimal places. [4 marks]

Answer .....

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C1 [4 marks]

Amir expands $\sqrt{4-x}$. He writes: "$\sqrt{4-x} = 4\left(1-\frac{x}{4}\right)^{\frac{1}{2}} = 4 - \frac{1}{2}x - \frac{1}{32}x^2$." Find his mistake and give the correct first three terms.

Answer .....

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C2 Find the first three terms of the expansion of $\dfrac{1}{5+2x}$ and state the values of x for which it is valid. Can the expansion be used when $x = 2$? When $x = 3$? Give a reason. [5 marks]

Answer .....

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C3 Raj writes: "$(2+3x)^{-1} = 2^{-1}(1+3x)^{-1} = \frac{1}{2} - \frac{3}{2}x + \frac{9}{2}x^2$." Find his mistake. Give the correct first three terms and the values of x for which they are valid. [4 marks]

Answer .....

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C4 (a) Find the first four terms of the expansion of $(4+x)^{\frac{3}{2}}$. (b) Use your expansion with $x = 0.2$ to estimate $4.2^{\frac{3}{2}}$. Compare with the value from a calculator. [5 marks]

Answer .....

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D1 $f(x) = \dfrac{1}{\sqrt{9-4x}}$

(a) Find the first four terms of the binomial expansion of $f(x)$, in ascending powers of x. Give each coefficient as a simplified fraction. [5 marks]

Answer .....

(b) Hence find the expansion of $(6-x)f(x)$, up to and including the term in x^2. [3 marks]

Answer .....

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(c) State the values of x for which the expansion is valid. Explain why it cannot be used to estimate $\frac{1}{\sqrt{21}}$. [2 marks]

Total for D1: 10 marks

D2

> Tom writes: "$(8+3x)^{-\frac{1}{3}} = 8^{-\frac{1}{3}}\left(1+\frac{3x}{8}\right)^{-\frac{1}{3}} = \frac{1}{2} - \frac{1}{8}x + \frac{1}{32}x^2$, valid for $|x| < 1$."

Tom has made two errors.

(a) Describe both errors. [2 marks]

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(b) Find the first three terms of the correct expansion of $(8 + 3x)^{-\frac{1}{3}}$. [4 marks]

Answer .....

(c) Use your expansion to estimate $\frac{1}{\sqrt[3]{9}}$, giving your answer to 3 decimal places. [2 marks]

Answer .....

Total for D2: 8 marks

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D3 Given that $(a+bx)^{-\frac{3}{2}} = \frac{1}{8} + px + \frac{15}{256}x^2 + qx^3 + \ldots$, where a, b, p and q are constants and $b < 0$, find the values of a, b, p and q, and state the values of x for which the expansion is valid. [9 marks]

Answer .....

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D4 $g(x) = \dfrac{1}{(2-5x)^2}$

(a) Find the first four terms of the binomial expansion of $g(x)$, in ascending powers of x. [5 marks]

Answer .....

(b) In the expansion of $(k+x)g(x)$, where k is a constant, there is no term in x^2. Find k. [3 marks]

Answer .....

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(c) State the values of x for which the expansion of $g(x)$ is valid. [2 marks]

Answer .....

Total for D4: 10 marks

D5 $h(x) = (16 + x)^{\frac{1}{4}}$

(a) Find the first three terms of the binomial expansion of $h(x)$, in ascending powers of x. [4 marks]

Answer .....

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(b) Use your expansion to estimate $\sqrt[4]{17}$, giving your answer to 5 decimal places. Find the percentage error of this estimate, to 2 significant figures. [3 marks]

Answer .....

(c) Explain why this expansion should not be used to estimate $\sqrt[4]{40}$. [2 marks]

Total for D5: 9 marks

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D6 $p(x) = \dfrac{1}{3+2x} + \dfrac{1}{3-2x}$

(a) Find the first three terms of the expansion of $\dfrac{1}{3+2x}$, in ascending powers of x. [3 marks]

Answer .....

(b) Hence show that, for small x, $p(x) \approx \dfrac{2}{3} + \dfrac{8}{27}x^2$. [3 marks]

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(c) State the values of x for which this is valid, and compare the approximation with the exact value of $p(0.3)$. [2 marks]

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The binomial expansion of $(a + bx)^n$, where n is a negative integer and $a > 0$, begins $\frac{1}{9} + \frac{2}{9}x + \frac{1}{3}x^2 + \ldots$ Find n, a and b. Find the coefficient of x^3 and the values of x for which the expansion is valid. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write the expansion as $a^n\left(1+ncx+\dfrac{n(n-1)}{2}c^2x^2+\dots\right)$ with $c=\dfrac{b}{a}$.

Hint 2 Divide each coefficient by the constant term $\dfrac{1}{9}$: you get $nc=2$ and $\dfrac{n(n-1)}{2}c^2=3$.

Hint 3 Substitute $c=\dfrac{2}{n}$ into the second equation. It becomes linear in n.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can take out a factor of a^n and expand $(a + bx)^n$ for negative or fractional n.	○ ○ ○
I can state the values of x for which an expansion of $(a + bx)^n$ is valid.	○ ○ ○
I can expand a product or quotient with a linear factor and find unknown constants.	○ ○ ○
I can use an expansion to approximate a root and judge its accuracy.	○ ○ ○

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