Binomial expansion with partial fractions

123 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-binomial-expansion-with-partial-fractions/

Date

A Do Now

Quick questions to start.

A1 Needed today [3 marks]

Express $\dfrac{x+7}{(x+1)(x-2)}$ in partial fractions.

Answer .....

A2 Needed today [2 marks]

Write down the first three terms of the expansions of $(1-x)^{-1}$ and of $(1+2x)^{-1}$.

Answer .....

A3 Common slip [2 marks]

Write $\dfrac{1}{x-3}$ in the form $k\left(1-\dfrac{x}{3}\right)^{-1}$.

Answer .....

A4 Common slip [2 marks]

One expansion is valid for $|x| < \dfrac{1}{2}$ and another for $|x| < 3$. For which values of x is their sum valid?

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Express $\dfrac{4-5x}{(1+x)(2-x)}$ in partial fractions. Hence find its expansion in ascending powers of x up to the term in x^3, and state the values of x for which it is valid.

Hint Expand $\dfrac{3}{1+x}$ and $\dfrac{2}{2-x} = \left(1-\dfrac{x}{2}\right)^{-1}$ separately, then subtract.

B1 [7 marks]

Express $\dfrac{x+5}{(1-x)(2+x)}$ in partial fractions. Hence find its expansion in ascending powers of x up to the term in x^3, and state the values of x for which it is valid.

Answer .....

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EXAMPLE 2

Given $\dfrac{1+3x}{(1-x)(1+x)^2} = \dfrac{1}{1-x} + \dfrac{1}{1+x} - \dfrac{1}{(1+x)^2}$, find its expansion up to the term in x^3.

Hint $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots$

B2 Express $\dfrac{3-4x+2x^2}{(1+x)(1-2x)^2}$ in the form $\dfrac{A}{1+x} + \dfrac{B}{1-2x} + \dfrac{C}{(1-2x)^2}$. Hence find its expansion up to the term in x^2 and state the values of x for which it is valid. [8 marks]

Answer .....

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EXAMPLE 3

Express $\dfrac{3x^2+4x-1}{(1-x)(2+x)}$ in the form $A+\dfrac{B}{1-x}+\dfrac{C}{2+x}$. Hence find its expansion up to the term in x^2.

Hint The top and bottom are both of degree 2, so there is a constant A. The x^2 terms give A: $3=-A$.

B3 Express $\dfrac{2x^2-x}{(1+x)(2-x)}$ in the form $A+\dfrac{B}{1+x}+\dfrac{C}{2-x}$. Hence find its expansion up to the term in x^3. [8 marks]

Answer .....

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EXAMPLE 4

Using $\dfrac{4-5x}{(1+x)(2-x)} = \dfrac{3}{1+x} - \dfrac{2}{2-x}$, find the coefficient of x^r in its expansion.

Hint The coefficient of x^r in $(1+x)^{-1}$ is $(-1)^r$, and in $\left(1-\dfrac{x}{2}\right)^{-1}$ it is $\left(\dfrac{1}{2}\right)^r$.

B4 Using $\dfrac{x+5}{(1-x)(2+x)} = \dfrac{2}{1-x} + \dfrac{1}{2+x}$, find the coefficient of x^r in its expansion. [4 marks]

Answer .....

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C1 Mia uses $\dfrac{x+4}{(x-2)(x+1)} = \dfrac{2}{x-2} - \dfrac{1}{x+1}$. She writes "$\dfrac{2}{x-2} = 2\left(1+\dfrac{x}{2}\right)^{-1} = 2 - x + \dfrac{1}{2}x^2$". Find her mistake and give the correct expansion of $\dfrac{x+4}{(x-2)(x+1)}$ up to the term in x^2. [6 marks]

Answer .....

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C2 $\dfrac{7}{(1-2x)(3+x)} = \dfrac{2}{1-2x} + \dfrac{1}{3+x}$. Sam says the expansion of this is valid for $|x| < 3$. Is he right? Give the correct range and say whether the expansion could be used with $x = 0.4$ and with $x = 0.6$. [5 marks]

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C3 $f(x) = \dfrac{3+3x}{(1+2x)(1-x)}$. (a) Express $f(x)$ in partial fractions and find its expansion up to the term in x^3. (b) Use it to estimate $f(0.05)$ and find the percentage error, to 2 significant figures. (c) Explain why the expansion should not be used for $f(0.6)$. [8 marks]

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C4 By writing $\dfrac{2+x}{(1-x)^2} = \dfrac{A}{1-x} + \dfrac{B}{(1-x)^2}$, find the coefficient of x^r in its expansion. Hence find the coefficient of x^5. [4 marks]

Answer .....

D Exam-style questions

Edexcel exam style

D1 $f(x) = \dfrac{6-11x}{(2+x)(1-3x)}$

(a) Express $f(x)$ in partial fractions. [3 marks]

Answer .....

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(b) Hence find the series expansion of $f(x)$, in ascending powers of x, up to and including the term in x^2. Give each coefficient as a simplified fraction. [5 marks]

Answer .....

(c) State, with a reason, the values of x for which the expansion is valid. [2 marks]

Total for D1: 10 marks

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D2

> Lily writes: "$\dfrac{3}{1-x} - \dfrac{2}{4+x} = 3(1-x)^{-1} - \dfrac{2}{4}(1+x)^{-1}$, and this is valid for $|x| < 4$."

Lily has made two errors.

(a) Describe both errors. [3 marks]

(b) Find the correct expansion up to and including the term in x^2. [5 marks]

Answer .....

Total for D2: 8 marks

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D3 Find the series expansion of $\dfrac{4x^2 - 3x + 2}{(1 + x)(1 - 2x)}$, in ascending powers of x, up to and including the term in x^3. State the values of x for which it is valid. [9 marks]

Answer .....

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D4 $g(x) = \dfrac{5}{(1-2x)(1+3x)}$

(a) Express $g(x)$ in partial fractions. [2 marks]

Answer .....

(b) Show that the coefficient of x^r in the expansion of $g(x)$ is $2^{r+1} + (-1)^r 3^{r+1}$. [4 marks]

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(c) Find the coefficient of x^4 and state the values of x for which the expansion is valid. [3 marks]

Answer .....

Total for D4: 9 marks

D5 $h(x) = \dfrac{10 + x}{(1 + x)(2 - x)^2}$

(a) Express $h(x)$ in the form $\dfrac{A}{1 + x} + \dfrac{B}{2 - x} + \dfrac{C}{(2 - x)^2}$. [4 marks]

Answer .....

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(b) Hence find the expansion of $h(x)$ up to and including the term in x^2. [4 marks]

Answer .....

(c) Use your expansion to estimate $h(0.1)$ and find the percentage error, to 2 significant figures. [2 marks]

Answer .....

Total for D5: 10 marks

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D6 $p(x) = \dfrac{1 + kx}{(1 - x)(1 + 2x)}$, where k is a constant.

(a) Show that $p(x) = \dfrac{1 + k}{3(1 - x)} + \dfrac{2 - k}{3(1 + 2x)}$. [3 marks]

(b) In the expansion of $p(x)$ there is no term in x^2. Find k and the coefficient of x^3. [5 marks]

Answer .....

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Show that the coefficient of x^r in the expansion of $\dfrac{1}{(1-2x)(1+x)}$ is $\dfrac{2^{r+1}+(-1)^r}{3}$. Prove that this is a whole number for every $r \geq 0$. Find the smallest r for which it is greater than 1000. [10 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Start with partial fractions:

$$\frac{1}{(1-2x)(1+x)} = \frac{A}{1-2x} + \frac{B}{1+x}.$$

Hint 2 For the whole number: $2 = 3 - 1$, so 2^{r+1} leaves the same remainder on division by 3 as $(-1)^{r+1}$.

Hint 3 The coefficient roughly doubles each time. Try $r = 10$ and $r = 11$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can split a fraction into partial fractions and expand each part.	○ ○ ○
I can expand fractions with a repeated factor and improper fractions.	○ ○ ○
I can state the values of x for which the whole expansion is valid.	○ ○ ○
I can find the general term and use the expansion to approximate.	○ ○ ○

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