

Cubic and quartic graphs

71 marks



Hints online, answers free
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mathswariz.uk/worksheets/a-level-cubic-and-quartic-graphs/

Date _____

A Do Now

Quick questions to start.

- | | |
|---|---|
| A1 Needed today [2 marks]
Factorise completely
$5x^3 - 45x$. | A2 Needed today [2 marks]
Expand and simplify
$(x - 2)(x + 3)(x + 1)$. |
|---|---|

Answer

Answer

- | | |
|--|---|
| A3 Last lesson [2 marks]
Find the set of values of x
for which $x^2 < 3x + 10$. | A4 From chapter 2 [2 marks]
Find the coordinates of
the turning point of
$y = x^2 - 8x + 19$. |
|--|---|

Answer

Answer (....., ...)

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Sketch $y = (x + 4)(x - 1)(3 - x)$, showing where it meets the axes.

Hint Put $y = 0$, then $x = 0$. The x^3 term is $x \times x \times (-x) = -x^3$.

B1 Sketch [3 marks]

$y = (x - 2)(x + 1)(x + 5)$, showing the coordinates of the points where it meets the axes.

EXAMPLE 2

Factorise $3x^3 + 12x^2 + 12x$ completely. Hence sketch $y = 3x^3 + 12x^2 + 12x$.

Hint Take out the common factor $3x$ first. A squared factor gives a repeated root: the curve touches the axis there.

B2 Factorise [4 marks]

$20x^2 - 4x^3 - 25x$ completely. Hence sketch $y = 20x^2 - 4x^3 - 25x$, showing where it meets the axes.

EXAMPLE 3

Sketch $y = -(x + 2)(x - 1)^2(x - 3)$, showing where it meets the axes.

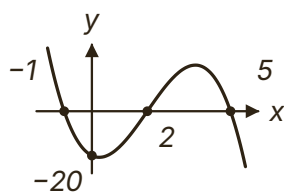
Hint A quartic. The x^4 coefficient is -1 , so both ends go down. The squared factor gives a point where the curve touches.

B3 Sketch [3 marks]

$y = (x + 3)(x + 1)(x - 2)^2$, showing the coordinates of the points where it meets the axes.

EXAMPLE 4

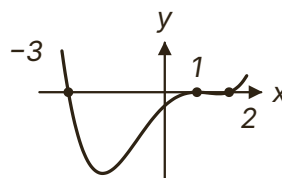
The sketch shows the curve $y = ax^3 + bx^2 + cx + d$. It crosses the x -axis at $x = -1$, $x = 2$ and $x = 5$, and the y -axis at $(0, -20)$. Find a , b , c and d .



Hint Write $y = a(x + 1)(x - 2)(x - 5)$ and use the y -intercept to find a .

B4 The sketch shows the [4 marks]

curve $y = x^4 + bx^3 + cx^2 + dx + e$. It crosses the x -axis at $x = -3$ and $x = 2$, and touches it at $x = 1$. Find b , c , d and e .



Answer

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- C1** Sketch the curve $y = (x + 1)(x - 2)(3 - x)(x + 4)$, showing the coordinates of the points where it meets the axes. [4 marks]

-
- C2** Show that the curve $y = (x + 2)(x^2 - 2x + 5)$ meets the x -axis at only one point. Sketch the curve, showing where it meets the axes. [4 marks]

-
- C3** Factorise $(x^2 + 2x - 15)(x^2 - 6x + 9)$ completely. Hence sketch $y = (x^2 + 2x - 15)(x^2 - 6x + 9)$, showing the coordinates of the points where it meets the axes. [5 marks]
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D1 $f(x) = (x + 4)(2x^2 - 2x) - 12x$

- (a)** Express $f(x)$ in the form $x(ax^2 + bx + c)$, where a , b and c are constants. [2 marks]

Answer

- (b)** Hence factorise $f(x)$ completely. [2 marks]

Answer

- (c)** Sketch the curve $y = f(x)$, showing the coordinates of the points where it meets the axes. [3 marks]

- (d)** Hence find the set of values of x for which $f(x) > 0$. [2 marks]

Answer

Total for D1: 9 marks

D2

Sam solves $(x - 4)(x + 1)^2 \leq 0$. He writes: "The curve is below the x -axis between its roots, so the answer is $-1 \leq x \leq 4$."

Sam is wrong.

- (a) Give a value of x that is not in Sam's answer but does satisfy $(x - 4)(x + 1)^2 \leq 0$, and explain what Sam has missed. [2 marks]

- (b) Sketch $y = (x - 4)(x + 1)^2$ and find, in set notation, the correct set of values of x . [4 marks]

Answer

Total for D2: 6 marks

D3 The curve C has equation $y = (x - 1)(x^2 + kx + 16)$, where k is a constant.

- (a) Find the set of values of k for which C meets the x -axis at exactly one point. [4 marks]

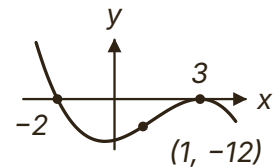
Answer

- (b) Given that $k = 8$, sketch C , showing the coordinates of the points where it meets the axes. [3 marks]

Total for D3: 7 marks

- D4** The sketch shows the curve C with equation $y = a(x + p)(x - q)^2$, where a , p and q are constants. C crosses the x -axis at $x = -2$, touches it at $x = 3$, and passes through $(1, -12)$.

- (a) Find the values of a , p and q . [3 marks]



Answer

- (b) Write the equation of C in the form $y = ax^3 + bx^2 + cx + d$. [2 marks]

Answer

- (c) The line $y = x - 3$ meets C at three points. Using your answer to part (a), find the exact x -coordinates of these points. [3 marks]

Answer

Total for D4: 8 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The curve $y = (x^2 - 4)(x^2 - k)$, where k is a constant, can meet the x -axis at two, three or four points. For each of these numbers, find the set of values of k that gives it. [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** The roots are $x = \pm 2$ and the solutions of $x^2 = k$. How many solutions can $x^2 = k$ have?
- Hint 2** $k < 0$: none. $k = 0$: one, $x = 0$. $k > 0$: two, $\pm\sqrt{k}$, which are ± 2 when $k = 4$.
- Hint 3** Two points: $k < 0$ or $k = 4$. Three: $k = 0$. Four: $k > 0, k \neq 4$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can sketch a cubic or quartic from its factors. ☐ ☐ ☐

I can show where a curve touches the axis. ☐ ☐ ☐

I can find an equation from a sketch. ☐ ☐ ☐

I can solve an inequality from a sketch. ☐ ☐ ☐