

Definite integrals and areas under curves

70 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-definite-integrals-and-areas/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] **A2** Last lesson [2 marks]
Solve $15 + 2x - x^2 = 0$. Find $\int \left(6x^2 - \frac{4}{x^3}\right) dx$.

Answer $x = \dots\dots$

Answer

- A3** From chapter 12 [2 marks] **A4** From chapter 1 [2 marks]
Find the stationary points of $y = x^3 - 12x$. Find the value of $9^{\frac{3}{2}} - 4^{\frac{3}{2}}$.

Answer

Answer

B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find the exact value of

$$\int_1^4 \left(3\sqrt{x} - \frac{2}{x^2}\right) dx.$$

Hint Integrate, then work out (value at 4) minus (value at 1).

- B1** Find the exact value of [2 marks]

$$\int_1^9 \frac{x-1}{\sqrt{x}} dx.$$

Answer

EXAMPLE 2

Given that A is a constant and

$\int_1^2 (3x^2 - Ax) dx = A^2$, show that there are two possible values of A , and find them.

Hint Treat A as a number. Integrate and use the limits to get an expression in A , then set it equal to A^2 .

B2

Given that k is a constant and

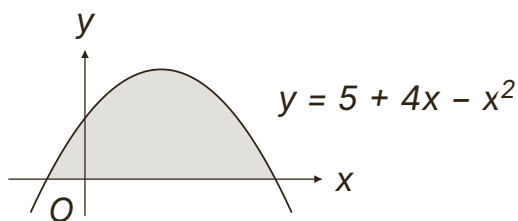
[4 marks]

$\int_0^3 (x^2 - kx) dx = k^2$, find the two possible values of k .

Answer $k = \dots\dots$

EXAMPLE 3

The sketch shows the curve $y = 5 + 4x - x^2$. Find the area of the shaded region, which is bounded by the curve and the x -axis.

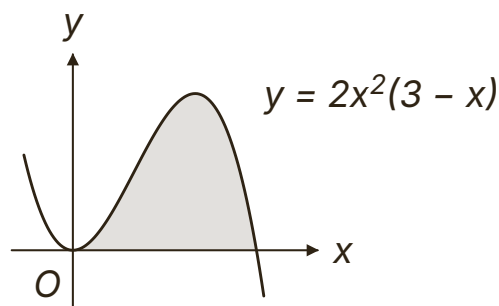


Hint The region starts and ends where the curve meets the x -axis. Find those x values first: they are the limits.

B3

The sketch shows $y = 2x^2(3 - x)$. Find the area of the shaded region, bounded by the curve and the x -axis.

[4 marks]



Answer

- C1** Given that $\int_1^k (3x^2 - 2) dx = 22$, where $k > 1$, find the value of k . [3 marks]

Answer $k = \dots\dots$

- C2** Find $\int_2^8 \left(2 - \frac{3}{\sqrt{x}}\right) dx$, giving your answer in the form $a + b\sqrt{2}$, [4 marks]
where a and b are integers.

Answer

- C3** Find the exact value of $\int_{-2}^{-1} \left(3x^2 + \frac{4}{x^3}\right) dx$. [3 marks]

Answer

D1 Sam writes: $\int_{-1}^2 (x^2 + 2x) dx = \left[\frac{x^3}{3} + x^2 \right]_{-1}^2 = \frac{8}{3} + 4 = \frac{20}{3}$ [5 marks]

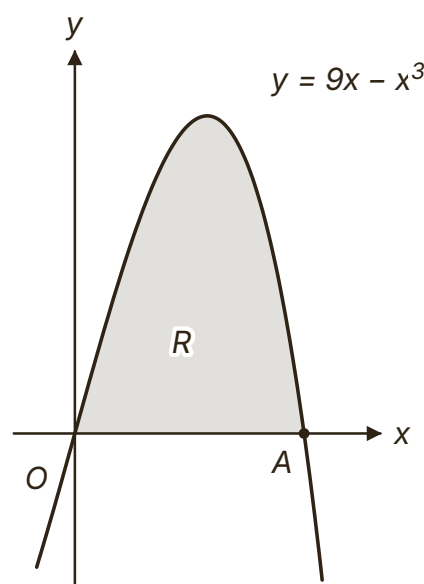
Sam is wrong. Explain his mistake, and find the correct value of the integral. Hence find the two values of the constant k for which

$$\int_{-1}^2 (x^2 + 2x + k) dx = 3k^2.$$

Answer

D2 The sketch shows part of the curve C with equation $y = 9x - x^3$. The curve meets the positive x -axis at A . The region R , shaded, is bounded by C and the positive x -axis.

- (a) Find the x -coordinate of A , and the exact coordinates of the maximum point of C in the region $x > 0$. [5 marks]



Answer

(b) Use calculus to find the area of R .

[4 marks]

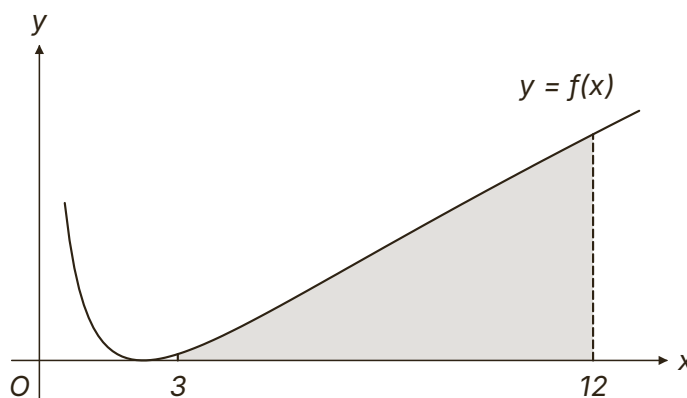
Answer

(c) The line $x = \sqrt{3}$ divides R into two parts. Hence show that the areas of the left part and the right part are in the ratio 5 : 4. [3 marks]

Total for D2: 12 marks

D3 $f(x) = \frac{(2\sqrt{x} - 3)^2}{\sqrt{x}}$, $x > 0$. The sketch shows part of the curve $y = f(x)$.

(a) Show that [2 marks]
 $f(x) = 4x^{\frac{1}{2}} - 12 + 9x^{-\frac{1}{2}}$.



(b) Find $\int f(x) dx$, giving each term in its simplest form.

[3 marks]

Answer

(c) Hence find the area of the shaded region, which is bounded by the curve, the x -axis and the lines $x = 3$ and $x = 12$. Give your answer in the form $a\sqrt{3} + b$, where a and b are integers, and explain why the definite integral gives the area.

[4 marks]

Answer

Total for D3: 9 marks

- D4** A cyclist starts from rest. Her speed, v metres per second, t seconds after she starts, is modelled by

$$v = kt(6 - t), \quad 0 \leq t \leq 6$$

where k is a constant. The distance she travels in the 6 seconds is 72 metres. The distance travelled is the area under the speed-time graph.

- (a)** Show that $k = 2$. Hence find her greatest speed. [5 marks]

- (b)** Find the distance she travels while her speed is at least 10 metres per second. [5 marks]

Answer

Total for D4: 10 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 $f(x) = x^2 + bx + c$, where b and c are constants. Given that

[6 marks]

$$\int_0^1 f(x) dx = 0 \quad \text{and} \quad \int_0^2 f(x) dx = 0,$$

find $\int_0^3 f(x) dx$.

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Each condition is an equation in b and c .
Two equations, two unknowns.
- Hint 2** $\frac{1}{3} + \frac{b}{2} + c = 0$ and $\frac{8}{3} + 2b + 2c = 0$.
- Hint 3** Halve the second: $\frac{4}{3} + b + c = 0$. Subtract the first: $1 + \frac{b}{2} = 0$, so $b = -2$ and $c = \frac{2}{3}$.
Then $\int_0^3 f(x) dx = 9 - 9 + 2$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can evaluate a definite integral exactly, including surd forms. ○ ○ ○

I can find an unknown constant or limit from a definite integral. ○ ○ ○

I can find the area between a curve and the x-axis. ○ ○ ○