

Differentiating x^n and sums of terms

72 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-differentiating-powers-of-x/

Date _____

A Do Now

Quick questions to start.

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| <p>A1 Needed today [2 marks]</p> <p>Write $\frac{x^2 + 5}{\sqrt{x}}$ as a sum of powers of x.</p> <p style="text-align: right;">Answer</p> | <p>A2 Last lesson [2 marks]</p> <p>$A(1, 3)$ lies on $y = x^2 + 2$. Find the gradient of the chord from A to the point on the curve with x-coordinate $1 + h$.</p> <p style="text-align: right;">Answer</p> |
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- | | |
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| <p>A3 From chapter 8 [2 marks]</p> <p>Find the coefficient of x^2 in the expansion of $(1 + 2x)^5$.</p> <p style="text-align: right;">Answer</p> | <p>A4 From chapter 2 [2 marks]</p> <p>Solve $3x^2 - 2x - 8 = 0$.</p> <p style="text-align: right;">Answer $x = \dots\dots$</p> |
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Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find $\frac{dy}{dx}$ when y equals: (a) $5x^4$ (b) $\frac{6}{x^2}$
(c) $4\sqrt[3]{x}$ (d) $\frac{x^5}{2x^2}$

Hint Write each one as ax^n first. Then multiply by the power and take 1 off the power.

B1 Find $\frac{dy}{dx}$ when y equals: [5 marks]

(a) $3x^7$ (b) $\frac{10}{x^5}$ (c) $8\sqrt[4]{x}$
(d) $\frac{x^2 \times x^4}{3x}$ (e) $\sqrt{16x^5}$

EXAMPLE 2

$y = 2x^3 - \frac{4}{x} + 3\sqrt{x} - 7$, $x > 0$. Find $\frac{dy}{dx}$, and find the gradient of the curve at the point (4, 126).

Hint Differentiate one term at a time. A constant term has gradient 0.

B2 $y = x^3 + \frac{16}{x^2} - 4\sqrt{x}$, [5 marks]

$x > 0$. Find $\frac{dy}{dx}$, and find the gradient of the curve at the point A(4, 57).

Answer gradient at A :

EXAMPLE 3

$f(x) = \frac{(2x-1)^2}{x}, x \neq 0$. (a) Write $f(x)$ in the form $Ax + B + Cx^{-1}$. (b) Find $f'(x)$. (c) Find the coordinates of the points on $y = f(x)$ where the gradient is 0.

Hint Expand the bracket, then divide every term by x . You cannot differentiate the top and the bottom separately.

B3

$$f(x) = \frac{(3x+2)^2}{x},$$

[6 marks]

$x \neq 0$. Find $f'(x)$, and find the coordinates of the points on $y = f(x)$ where the gradient is 5.

Answer (...,...) , (...,...)

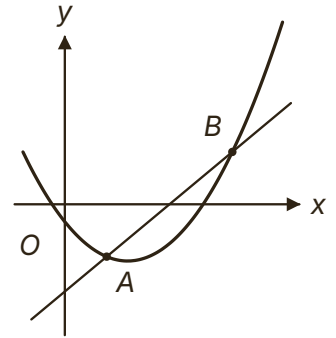
c Practice

A-Level

- C1** $f(x) = 3x^2 - 7x + 2$. (a) Find the gradient of $y = f(x)$ at the point $(2, 0)$. (b) Find the coordinates of the point on $y = f(x)$ at which the gradient is 11. [4 marks]

Answer (a) (b) (.....,.....)

- C2** The curve $y = x^2 - 3x - 1$ meets the line $y = 2x - 5$ at the points A and B . Find the gradient of the curve at A and at B . [4 marks]



Not drawn accurately

Answer at A : at B :

- C3** Given that $9y^2 = 4x^5$ and $y > 0$, find $\frac{dy}{dx}$. Hence find the gradient of the curve at the point where $x = 9$. [4 marks]

Answer $\frac{dy}{dx} = \dots\dots$ gradient $\dots\dots$

D Exam-style questions

Edexcel exam style

- D1** Priya differentiates $y = \frac{x^3 + 16}{2x}$, $x \neq 0$. She writes: "The top differentiates to $3x^2$ and the bottom to 2, so $\frac{dy}{dx} = \frac{3x^2}{2}$."

Priya is wrong.

- (a) Explain the mistake in Priya's method. [2 marks]

(b) Find $\frac{dy}{dx}$ correctly.

[3 marks]

Answer $\frac{dy}{dx} = \dots\dots$

(c) Hence find the coordinates of the point on the curve $y = \frac{x^3 + 16}{2x}$ [3 marks]
at which the gradient is 0.

Answer $(\dots\dots, \dots\dots)$

Total for D1: 8 marks

D2 The curve C has equation $y = ax^2 + \frac{b}{x}$, $x \neq 0$, where a and b are constants.
The gradient of C is -4 at the point where $x = 1$, and 6 at the point where $x = 2$.

(a) Show that $2a - b = -4$.

[2 marks]

(b) Hence find the value of a and the value of b .

[3 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

(c) The point P on C has x -coordinate -1 . Find the coordinates of P [3 marks]
and the gradient of C at P .

Answer $P(\dots\dots, \dots\dots)$ gradient $\dots\dots$

Total for D2: 8 marks

D3 $f(x) = \frac{k}{\sqrt{x}} + 3x$, $x > 0$, where k is a constant.

(a) Find $f'(x)$, in terms of k and x .

[2 marks]

Answer $f'(x) = \dots\dots$

(b) Given that $f'(2) = 2$, find the value of k , giving your answer in [3 marks]
the form $a\sqrt{2}$, where a is an integer.

Answer $k = \dots\dots$

(c) Hence find the exact value of $f(8)$.

[2 marks]

Answer

Total for D3: 7 marks

D4 $f(x) = (1 + 4x)^5$.

(a) Find the first three terms, in ascending powers of x , of the binomial expansion of $f(x)$, giving each term in its simplest form. [3 marks]

Answer + ... + ...

(b) Given that x is small, so that x^2 and higher powers of x in $f'(x)$ can be ignored, hence find an approximation for $f'(x)$ in the form $p + qx$. [2 marks]

Answer $f'(x) \approx \dots\dots$

(c) Use your answer to part (b) to estimate the gradient of $y = f(x)$ at the point where $x = 0.01$. [2 marks]

Answer

Total for D4: 7 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The curve C has equation $y = \frac{(\sqrt{x} + 2)^3}{x}$, $x > 0$. Show that there is exactly one point on C at which the gradient is 0, and find its coordinates. [6 marks]

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Expand $(\sqrt{x} + 2)^3$ as you would $(u + 2)^3$, with $u = \sqrt{x}$.
- Hint 2** Divide every term by x and write each as a power of x before you differentiate.
- Hint 3** Multiply $\frac{dy}{dx} = 0$ by $2x^2$ and put $u = \sqrt{x}$. One root of the cubic in u is a whole number.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can differentiate any power of x . ○ ○ ○

I can rewrite roots and fractions as powers. ○ ○ ○

I can find the gradient at a point. ○ ○ ○

I can find where the gradient takes a value. ○ ○ ○