Differentiating sin x, cos x, e^x and ln x

116 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-differentiating-sin-cos-exp-ln/

Date

A Do Now

Quick questions to start.

A1 Needed today [3 marks]

Differentiate $y = 4x^3 - \frac{2}{x} + 6\sqrt{x}$.

Answer .....

A2 Needed today [2 marks]

Expand $\sin(x + h)$ and $\cos(x + h)$.

Answer .....

A3 Common slip [2 marks]

Write $45°$ and $150°$ in radians. Say why the rule $\frac{\mathrm{d}}{\mathrm{d}x}(\sin x) = \cos x$ needs x in radians.

Answer .....

A4 Needed today [2 marks]

Write $\ln(5x)$ as a sum of two logarithms, and write e^{2x+3} as a constant times e^{2x}.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Prove, from first principles, that the derivative of $\cos x$ is $-\sin x$. You may use that, as $h \to 0$, $\dfrac{\sin h}{h} \to 1$ and $\dfrac{\cos h - 1}{h} \to 0$.

Hint Write the gradient of the chord from x to x + h, expand cos(x + h), and group the terms with cos x and with sin x.

B1 Prove, from first principles, that the derivative of $\sin 3x$ is $3\cos 3x$. You may use that, as $h \to 0$, $\dfrac{\sin h}{h} \to 1$ and $\dfrac{\cos h - 1}{h} \to 0$. [5 marks]

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EXAMPLE 2

The curve C has equation $y = 2\sin 3x + 4\cos\frac{x}{2}$. Find the exact gradient of C at the point where $x = \frac{\pi}{3}$.

Hint sin kx gives k cos kx and cos kx gives −k sin kx. Then put in x = π/3, in radians.

B2 [6 marks]

The curve C has equation $y = 5\cos 2x - 3\sin\frac{x}{3}$. Find an equation of the tangent to C at the point where $x = \frac{\pi}{2}$, giving exact values.

Answer .....

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EXAMPLE 3

Given $y = 4e^{\frac{x}{2}} - 6\ln(3x)$, find the exact value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = 2\ln 2$.

Hint e to the kx gives k e to the kx. ln(3x) = ln 3 + ln x, so its derivative is 1/x.

B3 Given $y = 2e^{3x} + 5\ln(2x)$, find the exact value of $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ when $x = \dfrac{1}{3}\ln 2$. [4 marks]

Answer .....

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EXAMPLE 4

By writing 2^{3x} as a power of e, show that $\dfrac{\mathrm{d}}{\mathrm{d}x}\left(2^{3x}\right) = 3\ln 2 \times 2^{3x}$. Find the gradient of $y = 2^{3x}$ at $x = 1$.

Hint Write $2 = e^{\ln 2}$, so $2^{3x} = e^{(3\ln 2)x}$. Then use the rule for e^{kx} with $k = 3\ln 2$.

B4 The curve $y = 5^{2x}$ passes through the point $P\left(\dfrac{1}{2}, 5\right)$. Find an equation of the tangent to the curve at P, giving exact values. [5 marks]

Answer .....

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EXAMPLE 5

Find the exact coordinates of the stationary point on the curve $y = e^{2x} - 6x$, and decide whether it is a maximum or a minimum.

Hint Set the derivative equal to 0 and solve for e^{2x} first. Then take logs.

B5 Find the exact coordinates of the stationary point on the curve $y = 4x - e^{\frac{x}{2}}$, and decide whether it is a maximum or a minimum. [6 marks]

Answer .....

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C1 Mia differentiates $y = 3\cos 4x + \ln(7x)$ and writes $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3\sin 4x + \dfrac{7}{x}$. Find her two mistakes and give the correct derivative. [4 marks]

Answer .....

C2 Liam is asked for the gradient of $y = \sin x^\circ$ at $x = 60$, where x is in degrees. He writes "$\cos 60^\circ = \dfrac{1}{2}$". Explain why Liam is wrong, and find the correct gradient. [4 marks]

Answer .....

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C3 Ben says the derivative of 3^x is $x \times 3^{x-1}$, "the same as for x^n". Explain why Ben is wrong. Find the exact gradient of $y = 3^x$ at $x = 2$ and compare it with Ben's value. [4 marks]

Answer .....

C4 Find the exact coordinates of the points on the curve $y = \sin x + \sqrt{3}\cos x$, $0 \leq x < 2\pi$, where the gradient is zero. [5 marks]

Answer .....

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C5 The tangent to the curve $y = \ln x$ at the point where $x = a$ passes through the origin. Find a and the equation of the tangent. [5 marks]

Answer .....

D Exam-style questions

Edexcel exam style

D1 The curve C has equation $y = e^{2x} - 5e^{x} + 3x$.

(a) Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$. [2 marks]

Answer .....

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(b) Hence find the exact coordinates of the two stationary points of C. [5 marks]

Answer .....

(c) Determine the nature of each stationary point. [4 marks]

Answer .....

Total for D1: 11 marks

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D2

Priya is finding the normal to $y = 4\cos 2x + 1$ at the point where $x = \frac{\pi}{6}$. She writes: "$\frac{\mathrm{d}y}{\mathrm{d}x} = 8\sin 2x$, so the gradient at $x = \frac{\pi}{6}$ is $4\sqrt{3}$. The point is $\left(\frac{\pi}{6}, 3\right)$, so the normal is $y - 3 = 4\sqrt{3}\left(x - \frac{\pi}{6}\right)$."

Priya is wrong.

(a) Identify her two mistakes. [3 marks]

(b) Find the correct equation of the normal in the form $y = mx + c$, giving m and c exactly. [4 marks]

Answer .....

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(c) Find the exact x-coordinate of the point where the normal meets the x-axis. [3 marks]

Answer .....

Total for D2: 10 marks

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D3 The tangent to the curve $y = e^{3x} - 9x$ at the point P is parallel to the line $y = 3x + 1$. Find the exact coordinates of P and the equation of this tangent, and show that the tangent meets the y-axis below the origin. [8 marks]

Answer .....

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D4 The temperature, $T\,^{\circ}\mathrm{C}$, of a cup of tea t minutes after it is made is modelled by $T = 20 + 60e^{-0.05t}$, $t \geq 0$.

(a) State the temperature of the tea when it is made. [2 marks]

Answer .....

(b) Find the rate of change of the temperature when $t = 10$. Give your answer to 3 significant figures, with units. [3 marks]

Answer .....

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(c) Find the exact time at which the tea is cooling at $1.5\,^{\circ}\mathrm{C}$ per minute, and give it to the nearest tenth of a minute. [4 marks]

Answer .....

(d) Give one feature of the model that suggests the room is at a temperature of 20 degrees, and one reason the model may not fit for large values of t. [2 marks]

Total for D4: 11 marks

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D5 The function f is defined by $\mathrm{f}(x) = 2x - \ln(4x)$, $x > 0$.

(a) Find $\mathrm{f}'(x)$. [2 marks]

Answer .....

(b) Find the exact coordinates of the stationary point of $y = \mathrm{f}(x)$. [3 marks]

Answer .....

(c) Show that this stationary point is a minimum. [2 marks]

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(d) Hence show that $\ln(4x) \leq 2x - 1 + \ln 2$ for all $x > 0$. [3 marks]

Total for D5: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The line $y = mx$ is a tangent to the curve $y = \ln(2x)$. Find the exact value of m. Hence find the set of values of k for which the equation $\ln(2x) = kx$ has exactly two real solutions. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Let the tangent touch the curve at $x = a$. Write its equation using the gradient $\frac{1}{a}$.

Hint 2 The tangent passes through $(0, 0)$. Put that in to find a, then m.

Hint 3 Sketch $y = \ln(2x)$ and the lines $y = kx$ through the origin. How many crossings for k below m, equal to m, above m, and for $k \leq 0$?

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can prove the derivative of sin kx or cos kx from first principles.	○ ○ ○
I can differentiate sin kx, cos kx, e to the kx, ln kx and a to the x, with x in radians.	○ ○ ○
I can find gradients, tangents and normals to curves with these functions, giving exact answers.	○ ○ ○
I can find and classify stationary points of curves with exponentials and logarithms.	○ ○ ○

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