Differentiating trigonometric functions

117 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-differentiating-trigonometric-functions/

Date

A Do Now

Quick questions to start.

A1 Last lesson [2 marks]

Differentiate $y = xe^{3x}$.

Answer .....

A2 Needed today [2 marks]

Given that $\tan x = \frac{3}{4}$ and x is acute, find the exact value of $\sec x$.

Answer .....

A3 Needed today [3 marks]

Write $y = \frac{1}{\sin x}$ as $(\sin x)^{-1}$ and use the chain rule to find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

Answer .....

A4 Common slip [2 marks]

Sam writes that the derivative of $\cos 3x$ is $3 \sin 3x$. Explain his mistake and give the correct derivative.

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Prove that $\frac{\mathrm{d}}{\mathrm{d}x}(\tan x) = \sec^2 x$.

Hint Write tan x as sin x over cos x and use the quotient rule. Then use sin squared plus cos squared equals 1.

B1 Prove that $\frac{\mathrm{d}}{\mathrm{d}x}(\cot x) = -\mathrm{cosec}^2 x$. [4 marks]

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EXAMPLE 2

Show that the derivative of $\sec x$ is $\sec x \tan x$.

Hint Write sec x as cos x to the power minus 1 and use the chain rule.

B2 Given $y = \operatorname{cosec} 3x$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and the exact gradient of the curve at the point where $x = \dfrac{\pi}{12}$. [4 marks]

Answer .....

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EXAMPLE 3

Given $y = x \tan 2x$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$.

Hint Product rule, with the chain rule giving 2 sec squared 2x for the tan.

B3 The curve C has equation $y = e^{2x} \sec x$, $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$. Find an equation of the tangent to C at the point where $x = 0$. [5 marks]

Answer .....

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EXAMPLE 4

Given that $x = \sin y$, $-\frac{\pi}{2} < y < \frac{\pi}{2}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of x.

Hint Find dx/dy, turn it upside down, then write cos y in terms of x. In this range cos y is positive.

B4 Given that $x = \tan y$, $-\frac{\pi}{2} < y < \frac{\pi}{2}$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{1+x^2}$. [4 marks]

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EXAMPLE 5

Find the exact x-coordinates of the stationary points of $y = 4x - \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$.

Hint Set 4 minus sec squared x equal to 0, so cos squared x is one quarter. Take both signs of the square root.

B5 The curve C has equation $y = \cot x + 2x$, $0 < x < \pi$. Find the exact x-coordinates of the stationary points of C and decide the nature of each. [5 marks]

Answer .....

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C1 Tom differentiates $y = \tan^3 2x$ and writes $\dfrac{\mathrm{d}y}{\mathrm{d}x} = 3\tan^2 2x \sec^2 2x$. [4 marks]

Explain his mistake, then find the gradient of the curve at the point where $x = \dfrac{\pi}{8}$.

Answer .....

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C2 The curve C has equation $y = \ln(\sec x)$, $0 < x < \frac{\pi}{2}$. Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = \tan x$, and find an equation of the tangent to C at the point where $x = \frac{\pi}{4}$, in exact form. [5 marks]

Answer .....

C3 Given that $x = \cot y$, $0 < y < \pi$, show that $\frac{\mathrm{d}y}{\mathrm{d}x} = -\frac{1}{1 + x^2}$. [4 marks]

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C4 Given $y = \dfrac{\sin x}{1 + \cos x}$, find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ in its simplest form. [5 marks]

Answer .....

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C5 Find the exact coordinates of the stationary point on the curve $y = 2\sec x - \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$. [6 marks]

Answer .....

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D Exam-style questions

Edexcel exam style

D1

> Kai says: "The derivative of $\sec^2 x$ is $2\sec x \tan x$, because the derivative of $\sec x$ is $\sec x \tan x$."

Kai is wrong.

(a) Explain his mistake and find the correct derivative of $\sec^2 x$. [3 marks]

Answer .....

(b) Use the identity $\tan^2 x = \sec^2 x - 1$ to check your answer, by differentiating $\tan^2 x$. [3 marks]

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(c) Find the exact gradient of the curve $y = \sec^2 x$ at the point where $x = \dfrac{\pi}{3}$. [3 marks]

Answer .....

Total for D1: 9 marks

D2 The curve C has equation $y = e^{-x}\tan x$, $-\dfrac{\pi}{2} < x < \dfrac{\pi}{2}$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = e^{-x}(\tan^2 x - \tan x + 1)$. [3 marks]

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(b) Hence show that C has no stationary points. [4 marks]

(c) Find an equation of the tangent to C at the origin. [2 marks]

Answer .....

(d) Find the exact gradient of C at the point where $x = \frac{\pi}{4}$. [2 marks]

Answer .....

Total for D2: 11 marks

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D3 Find an equation of the normal to the curve $y = \operatorname{cosec} x$ at the point where $x = \dfrac{\pi}{6}$, in the form $y = mx + c$, where m and c are exact. [6 marks]

Answer .....

D4 The curve C has equation $x = \sec y$, $0 < y < \dfrac{\pi}{2}$.

(a) Show that $\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{x\sqrt{x^2 - 1}}$. [4 marks]

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(b) Find an equation of the tangent to C at the point where $y = \frac{\pi}{3}$ [3 marks]

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Answer .....

(c) Explain why the gradient of C is positive at every point. [2 marks]

Total for D4: 9 marks

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D5 A straight rod touches the inside corner where a corridor 3 m wide meets a corridor 4 m wide at right angles, and its ends touch the two outer walls. When the rod makes an angle θ with the outer wall of the wider corridor, its length is $L = 3\sec\theta + 4\operatorname{cosec}\theta$, $0 < \theta < \dfrac{\pi}{2}$. The shortest such rod gives the longest rod that can be carried round the corner flat.

(a) Find $\dfrac{\mathrm{d}L}{\mathrm{d}\theta}$. [2 marks]

Answer .....

(b) Show that L is stationary when $\tan^3\theta = \dfrac{4}{3}$. [3 marks]

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(c) Find this value of θ, in radians to 3 significant figures. [2 marks]

Answer .....

(d) Find the length of the longest rod that can be carried round the corner flat, to 3 significant figures. [2 marks]

Answer .....

(e) Show that this value of θ gives the least value of L. [2 marks]

Total for D5: 11 marks

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D6 The curve C has equation $y = \ln(\tan x)$, $0 < x < \frac{\pi}{2}$.

(a) Show that $\frac{\mathrm{d}y}{\mathrm{d}x} = 2\,\mathrm{cosec}\,2x$. [4 marks]

(b) Find the least value of the gradient of C, and the exact value of x at which it occurs. [3 marks]

Answer .....

Total for D6: 7 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The curve C has equation $y = \sec x + k \tan x$, $-\frac{\pi}{2} < x < \frac{\pi}{2}$, where k is a constant. Show that C has a stationary point exactly when $-1 < k < 1$, that this stationary point is always a minimum, and that the least value of y is $\sqrt{1 - k^2}$. [9 marks]

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Differentiate, then write the derivative over the single denominator $\cos^2 x$. When is the top zero?

Hint 2 For which values of k does sin x equal minus k for an x strictly between minus a right angle and a right angle?

Hint 3 At the stationary point $\sin x = -k$. Find $\cos x$ in terms of k, then write y over $\cos x$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can differentiate tan kx, sec kx, cosec kx and cot kx. ○ ○ ○

I can prove these results with the quotient rule or the chain rule. ○ ○ ○

I can combine them with the chain, product and quotient rules. ○ ○ ○

I can find dy/dx in terms of x when x is a trigonometric function of y. ○ ○ ○

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