# Double-angle formulae

113 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-double-angle-formulae/

Date

## A Do Now

Quick questions to start.

**A1** Needed today [2 marks]

Use $\cos(A+B) = \cos A \cos B - \sin A \sin B$ to write $\cos(x+x)$ in terms of $\cos x$ and $\sin x$.

Answer .....

**A2** Needed today [2 marks]

$\cos\theta = \frac{5}{13}$ and $\theta$ is acute. Find the exact values of $\sin\theta$ and $\tan\theta$.

Answer .....

**A3** Common slip [2 marks]

Show that $\sin(2 \times 30^\circ)$ is not equal to $2\sin 30^\circ$.

Answer .....

**A4** From chapter 6 [3 marks]

Solve $2\sin^2 x - \sin x - 1 = 0$ for $0^\circ \leq x < 360^\circ$.

Answer .....



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## B Example, then your turn



Your teacher works through each example with you. Then try the one beside it.

**EXAMPLE 1**

$\cos A = \frac{4}{5}$ and $A$ is acute. Find the exact values of $\sin 2A$ and $\cos 2A$.

**Hint** Find sin A first, then use sin 2A = 2 sin A cos A.

**B1** [5 marks]

$\sin A = -\frac{5}{13}$ and $\pi < A < \frac{3\pi}{2}$. Find the exact values of $\sin 2A$, $\cos 2A$ and $\tan 2A$.

Answer .....



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**EXAMPLE 2**

Solve $\sin 2x = \cos x$ for $0° \leq x < 360°$.

**Hint** Write sin 2x as 2 sin x cos x and factorise. Do not divide by cos x.

**B2** Solve $\sin 2x + \sin x = 0$ for $0° \leq x < 360°$. [5 marks]

Answer .....


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**EXAMPLE 3**

Solve $\cos 2x = \sin x$ for $0° \leq x < 360°$.

**Hint** Choose the form of cos 2x that leaves only sin x.

**B3** Solve $\cos 2x + 3\cos x + 2 = 0$ for $0 \leq x < 2\pi$. [5 marks]

Answer .....


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**EXAMPLE 4**

Show that $2\sin^2 x = 1 - \cos 2x$.

Hence find the exact value of $\sin^2 15°$.

**Hint** Start from cos 2x = 1 − 2 sin²x.

**B4** Show that $\cos^4 x - \sin^4 x = \cos 2x$. [4 marks]

Hence find the exact value of $\cos^4 \frac{\pi}{12} - \sin^4 \frac{\pi}{12}$.



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**EXAMPLE 5**

$\tan A = \frac{1}{2}$. Find the exact value of $\tan 2A$.

**Hint** Use tan 2A = 2 tan A divided by (1 – tan²A).

**B5** [5 marks]

$A$ is acute and $\tan 2A = \frac{3}{4}$. Find the exact value of $\tan A$, and the exact value of $\tan 4A$.

Answer .....



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**C1** Leo solves $\sin 2x = \sqrt{2}\sin x$ for $0^\circ \le x < 360^\circ$ like this: [4 marks]

$$2\sin x\cos x = \sqrt{2}\sin x, \quad \cos x = \frac{\sqrt{2}}{2}, \quad x = 45^\circ, 315^\circ.$$

Explain the mistake, and give all the solutions.

Answer .....

**C2** Use $\cos 2A = 2\cos^2 A - 1$ with $A = 22.5^\circ$ to find the exact value of $\cos 22.5^\circ$. Find the exact value of $\sin 22.5^\circ$ in the same way. [4 marks]

Answer .....


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**C3** Write $\cos 3x$ as $\cos(2x + x)$ and prove that $\cos 3x = 4\cos^3 x - 3\cos x$. [5 marks]

**C4** Show that $\dfrac{1 - \cos 2x}{\sin 2x} = \tan x$. Hence find the exact value of $\tan 15^\circ$. [5 marks]


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**C5** Solve $\cos 2x = 5\sin x - 2$ for $0° \leq x < 360°$. [4 marks]

Answer .....

## D Exam-style questions

Edexcel exam style

### D1

**(a)** Prove that $\sin 3x = 3\sin x - 4\sin^3 x$. [4 marks]


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**(b)** Hence solve $\sin 3x = 2\sin x$ for $0 \leq x < 2\pi$. [5 marks]

Answer .....

**Total for D1: 9 marks**

**D2**

> Ava writes: "$\cos 2x = 1 - 2\sin x$, so $1 - 2\sin x + \sin x = 0$, which gives $\sin x = 1$ and $x = 90^\circ$ only."

Ava is asked to solve $\cos 2x + \sin x = 0$ for $0^\circ \leq x < 360^\circ$.

**(a)** Explain the mistake in Ava's working. [2 marks]


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**(b)** Solve the equation correctly. [5 marks]

Answer .....

**Total for D2: 7 marks**

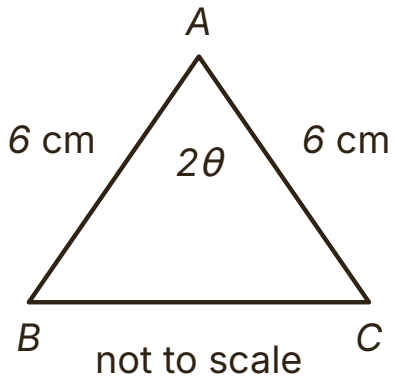

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**D3** Solve $2\sin 2\theta = 3\tan\theta$ for $0° \leq \theta < 360°$. [7 marks]

Answer .....

**D4** In triangle $ABC$, $AB = AC = 6$ cm and angle $BAC = 2\theta$, where $\theta$ is acute.

**(a)** Show that the area of the triangle is $18\sin 2\theta$ cm². [2 marks]





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**(b)** Show that $BC = 12\sin\theta$ cm. [3 marks]

**(c)** Given that $\cos\theta = \frac{4}{5}$, find the exact area of the triangle and the exact length of $BC$. [4 marks]

Answer .....

**Total for D4: 9 marks**



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**D5**

**(a)** Show that $\dfrac{\sin 2x}{1+\cos 2x} = \tan x$, for $\cos 2x \neq -1$. [3 marks]

**(b)** Hence find the exact value of $\tan 67.5°$. [3 marks]

Answer .....


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**(c)** Hence solve $\dfrac{\sin 2x}{1+\cos 2x} = \sqrt{3}$ for $0^\circ < x < 180^\circ$, $x \neq 90^\circ$. [3 marks]

Answer .....

**Total for D5: 9 marks**

## D6

**(a)** Show that $8\sin^4 x = 3 - 4\cos 2x + \cos 4x$. [5 marks]


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**(b)** Hence find the exact value of $\sin^4 \frac{\pi}{8}$. [2 marks]

Answer .....

**(c)** State the greatest value of $3 - 4\cos 2x + \cos 4x$ and the value of $x$ in $0 \leq x < \pi$ where it occurs. [2 marks]

Answer .....

**Total for D6: 9 marks**


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## E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

**E1** Use $5 \times 36^\circ = 180^\circ$ to find the exact value of $\cos 36^\circ$. Hence find the exact value of $\cos 72^\circ$. [8 marks]



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## Extension hints

Cover these with paper. Uncover one at a time.

**E1**

**Hint 1** With $\theta = 36°$, $2\theta$ and $3\theta$ add to $180°$. What does that say about $\sin 2\theta$ and $\sin 3\theta$?

**Hint 2** Write $\sin 2\theta = \sin 3\theta$ using $\sin 3\theta = 3\sin\theta - 4\sin^3\theta$, then divide by $\sin\theta$.

**Hint 3** You should get a quadratic in $\cos\theta$. Keep the positive root, then use $\cos 2A = 2\cos^2 A - 1$.

## How did it go?

Colour one circle on each line: not yet, nearly, yes.

| | |
|---|---|
| I can find sin 2A, cos 2A and tan 2A from one ratio of A, choosing signs from the quadrant. | ○ ○ ○ |
| I can choose the form of cos 2A that leaves one trigonometric function, and solve the equation. | ○ ○ ○ |
| I can use cos 2A to find exact values of half angles such as cos 22.5°. | ○ ○ ○ |
| I can rewrite powers such as $\sin^4 x$ in terms of cos 2x and cos 4x, and prove identities. | ○ ○ ○ |


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