

Equations of straight lines

70 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-equations-of-straight-lines/

Date _____

A Do Now

Quick questions to start.

- | | |
|---|--|
| <p>A1 [2 marks] Needed today</p> <p>Make y the subject of
$5x - 3y + 12 = 0$.</p> | <p>A2 [2 marks] GCSE Higher</p> <p>Solve the simultaneous
equations $y = 2x - 1$ and
$3x + 2y = 19$.</p> |
|---|--|

Answer $y = \dots\dots$

Answer $x = \dots\dots$ $y = \dots\dots$

- | | |
|--|---|
| <p>A3 [2 marks] From chapter 1</p> <p>Simplify $\frac{18}{\sqrt{6}} + \sqrt{54}$.</p> | <p>A4 [2 marks] Last lesson</p> <p>$y = f(x)$ has a maximum
point at $(2, 5)$. Write down
the maximum point of
$y = f(x - 3) + 1$.</p> |
|--|---|

Answer

Answer $(\dots\dots, \dots\dots)$

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Show that the points $A(-1, -4)$, $B(2, 2)$ and $C(7, 12)$ are collinear.

Hint Equal gradients for AB and BC , with B shared, means one line.

B1 Show that $P(3a, -a)$, $Q(a, 3a)$ and $R(-2a, 9a)$ are collinear, where $a \neq 0$. [4 marks]

EXAMPLE 2

The line L has equation $4x - 6y + 9 = 0$. Find its gradient and where it crosses each axis.

Hint Rearrange into the form $y = mx + c$.
On the x -axis $y = 0$.

B2 The line L has equation $6x + 4y - 15 = 0$. Find its gradient and where it crosses each axis. [4 marks]

Answer

EXAMPLE 3

Find an equation of the line through $(-3, 4)$ and $(5, -2)$, in the form $ax + by + c = 0$ with a , b and c integers.

Hint Find the gradient first. Then use $y - y_1 = m(x - x_1)$ and clear the fraction.

B3 Find an equation of the line through $(-4, -1)$ and $(2, 3)$, in the form $ax + by + c = 0$ with a , b and c integers. [4 marks]

Answer

c Practice

A-Level

C1 The lines $y = 3x - 11$ and $2x + 5y - 13 = 0$ meet at the point A . The point B has coordinates $(-3, 6)$. Find an equation of the line through A and B . Give your answer in the form $ax + by + c = 0$, where a , b and c are integers. [4 marks]

Answer

- C2** The line $5x + 2y - 30 = 0$ crosses the x -axis at the point A . The line l [3 marks]
passes through A and has gradient $-\frac{3}{4}$. Find an equation of l in the
form $ax + by + c = 0$, where a , b and c are integers.

Answer

- C3** The line with gradient $-\frac{2}{5}$ that crosses the y -axis at $(0, -3)$ has [2 marks]
equation $ax + 10y + c = 0$. Find the value of a and the value of c .

Answer $a = \dots\dots$ $c = \dots\dots$

- C4** The line through $(-6, 5)$ and $(3, -1)$ crosses the x -axis at P and the y [4 marks]
-axis at Q . Find the coordinates of P and of Q .

Answer $P(\dots\dots, 0)$ $Q(0, \dots\dots)$

D1 Amira is asked for an equation of the line through $(4, -1)$ and $(-2, 3)$. She writes: $m = \frac{3 - (-1)}{4 - (-2)} = \frac{4}{6} = \frac{2}{3}$, so $y + 1 = \frac{2}{3}(x - 4)$, giving $2x - 3y - 11 = 0$.

Amira is wrong.

(a) Show that Amira's line does not pass through $(-2, 3)$, and explain [2 marks]
her mistake.

(b) Find a correct equation of the line, in the form $ax + by + c = 0$, [3 marks]
where a , b and c are integers.

Answer

(c) The point $(k, 2k)$ lies on the line. Find the value of k . [3 marks]

Answer $k = \dots\dots$

Total for D1: 8 marks

D2 The line l_1 passes through the points $A(-4, 11)$ and $B(6, -4)$.

- (a)** Find an equation of l_1 [3 marks]
in the form
 $ax + by + c = 0$, where
 a, b and c are integers.

Answer

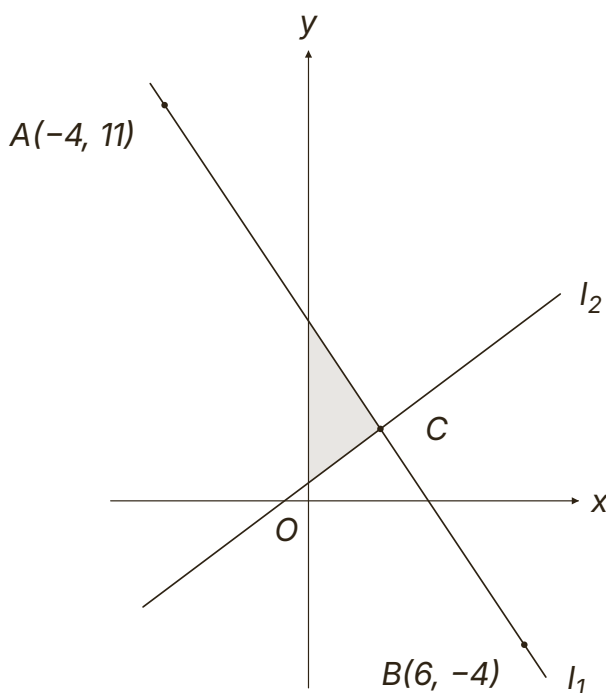
- (b)** The line l_2 has equation [3 marks]
 $3x - 4y + 2 = 0$. The
lines l_1 and l_2 meet at
 C . Find the coordinates
of C .

Answer (....., ...

- (c)** Hence find the area of the triangle formed by l_1 , l_2 and the y -axis. [4 marks]

Answer

Total for D2: 10 marks



Not drawn accurately

- D3** The straight line l passes through the points $A(2, \sqrt{3})$ and $B(3 + \sqrt{3}, 4 + \sqrt{3})$. Find the exact x -coordinate of the point where l crosses the x -axis. Give your answer in the form $p + q\sqrt{3}$, where p and q are rational numbers. [6 marks]

Answer $x = \dots\dots$

- D4** The straight line l passes through $A(1, p)$ and $B(p, 9)$, where p is a constant and $p \neq 1$. The gradient of l is p .

(a) Show that $p^2 = 9$. [3 marks]

(b) Hence find the two possible equations of l . [3 marks]

Answer $y = \dots\dots$ or $y = \dots\dots$

Total for D4: 6 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** A straight line passes through $P(2, 3)$ and meets the positive x -axis at A and the positive y -axis at B . Find the smallest possible area of triangle OAB , where O is the origin, and the equation of the line that gives it. [7 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Let A be $(a, 0)$ and B be $(0, b)$. The area is $\frac{1}{2}ab$.
- Hint 2** P is on $\frac{x}{a} + \frac{y}{b} = 1$, so $3a + 2b = ab$.
- Hint 3** $(3a - 2b)^2 \geq 0$ gives $(3a + 2b)^2 \geq 24ab$, so $ab \geq 24$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the gradient through two points. ☐ ☐ ☐

I can find a line's equation from a point and a gradient, or two points. ☐ ☐ ☐

I can write a line as $ax + by + c = 0$ and read off its gradient. ☐ ☐ ☐

I can find where lines meet each other and the axes. ☐ ☐ ☐