

Expanding brackets and factorising

77 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-expanding-brackets-and-factorising/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] Factorise $x^2 - 2x - 15$.
- A2** Needed today [2 marks] Expand and simplify $(3x + 2)(x - 5)$.

Answer

Answer

- A3** Last lesson [2 marks] Evaluate $16^{-\frac{3}{4}}$.
- A4** Last lesson [2 marks] Write $\frac{4x^2 + 3}{\sqrt{x}}$ in the form $ax^p + bx^q$.

Answer

Answer

B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Expand and simplify
 $(x + 2)(2x - y + 3)$.

Hint $2 \times 3 = 6$ products, then collect like terms.

- B1** Expand and simplify [3 marks]
 $(3x - y)(x + 4y - 2)$.

Answer

EXAMPLE 2

Expand and simplify

$$(x - 3)(2x + 5)(x + 1).$$

Hint Multiply two of the brackets first, then multiply the quadratic by the third bracket.

B2 Expand and simplify [4 marks]

$$(2x + 3)(x - 2)(3x - 1).$$

Answer

EXAMPLE 3Factorise $6x^2 + 7x - 20$.

Hint Find two numbers that multiply to $ac = 6 \times (-20) = -120$ and add to 7. Split the $7x$ with them, then factorise in pairs.

B3 Factorise (a) [3 marks]

$$10x^2 - 11x - 6 \quad (b)$$

$$25a^2 - 81b^2$$

Answer (a) (b)

EXAMPLE 4

Factorise completely (a)

$$3x^3 - 12x^2 - 36x \quad (b) 18x - 8x^3$$

Hint Take out the highest common factor first. Then factorise what is left in the bracket, if it will go.

B4 Factorise completely (a) [4 marks]

$$4x^3 + 2x^2 - 30x \quad (b)$$

$$75x - 3x^3$$

Answer (a) (b)

EXAMPLE 5

Write $x^4 - 34x^2 + 225$ as the product of four linear factors.

Hint Think of it as a quadratic in x^2 : $y^2 - 34y + 225$ with $y = x^2$. Factorise that, then each bracket is a difference of two squares.

B5 Factorise completely [4 marks]

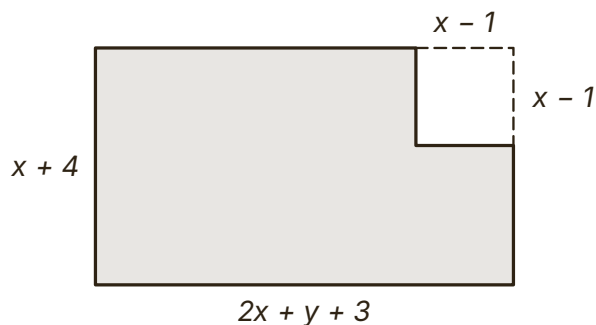
$$9x^4 - 37x^2 + 4.$$

Answer

c Practice

A-Level

- C1** A square of side $(x - 1)$ cm is cut from the corner of a rectangle that is $(2x + y + 3)$ cm long and $(x + 4)$ cm wide. Find an expanded and simplified expression for the area that is left, shaded in the diagram. [3 marks]



Answer

Not drawn accurately

- C2** Factorise $12 + 5x - 2x^2$. [2 marks]

Answer

- C3** Given that $(ax + 1)(x + b)(x - 2) \equiv 3x^3 + px^2 + qx + 8$, where a, b, p and q are constants, find the values of a, b, p and q . [4 marks]

Answer $a = \dots$ $b = \dots$ $p = \dots$ $q = \dots$

- C4** Factorise completely $2x^5 - 32x$. [3 marks]

Answer

- C5** Simplify fully $\frac{10x^2 + 13x - 3}{25x^2 - 1}$. [3 marks]

Answer

D1 In this question you must show all stages of your working. Solutions relying entirely on calculator technology are not acceptable.

(a) Factorise $10x^2 + 9x - 7$. [2 marks]

Answer

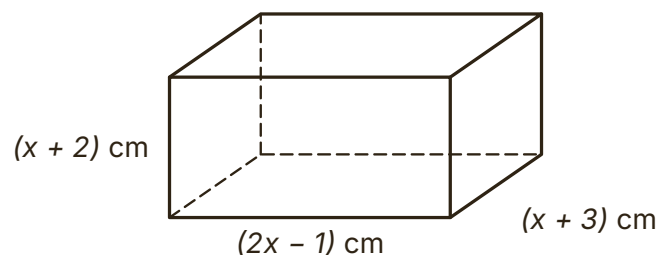
(b) Hence, without a calculator, write 9263 as the product of two prime numbers. [4 marks]

Answer $9263 = \dots \times \dots$

Total for D1: 6 marks

D2 A cuboid has edges of length $(x + 3)$ cm, $(2x - 1)$ cm and $(x + 2)$ cm, where $x > \frac{1}{2}$.

(a) Show that the volume of the cuboid is $(2x^3 + 9x^2 + 7x - 6)$ cm³. [3 marks]



Not drawn accurately

- (b) The total surface area of the cuboid is $(10x^2 + 26x + 2)$ cm², and [4 marks]
it is equal to 94 cm². Find the value of x and hence the volume of
the cuboid.

Answer $x = \dots\dots$ volume = $\dots\dots$ cm³

Total for D2: 7 marks

D3

Priya is told that $2x^3 + x^2 - 25x + 12 \equiv (x + 4)(2x^2 + bx + c)$. She writes:
"The x^2 terms give $b = 1$. The constant terms give $4c = 12$, so $c = 3$."

Priya has made a mistake.

- (a) Explain the mistake Priya has made. [1 mark]

- (b) Find the values of b and c , and use the x terms to check them. [3 marks]

Answer $b = \dots\dots$ $c = \dots\dots$

- (c) Hence factorise $2x^3 + x^2 - 25x + 12$ completely. [2 marks]

Answer

(d) Hence solve $2x^3 + x^2 - 25x + 12 = 0$.

[2 marks]

Answer

Total for D3: 8 marks

D4 $f(x) = 4x^4 - 17x^2 + 4$

(a) Write $f(x)$ as the product of four linear factors.

[4 marks]

Answer

(b) Hence solve $f(x) = 0$.

[2 marks]

Answer

(c) Hence solve $4^{2y+1} - 17 \times 4^y + 4 = 0$.

[3 marks]

Answer $y = \dots\dots$ or $y = \dots\dots$

Total for D4: 9 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Find every positive integer n for which $n^4 + 4$ is a prime number, [6 marks]
and prove that there are no others.

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** $n^4 + 4$ has no obvious factors, but it is close to a perfect square. Add and subtract a term to make a difference of two squares.
- Hint 2** $n^4 + 4 = (n^2 + 2)^2 - 4n^2 = (n^2 + 2)^2 - (2n)^2$.
- Hint 3** So $n^4 + 4 = (n^2 + 2n + 2)(n^2 - 2n + 2)$. The second factor is $(n - 1)^2 + 1$, which is 1 only when $n = 1$ and at least 2 otherwise. So for $n \geq 2$ both factors are at least 2 and $n^4 + 4$ is not prime. For $n = 1$, $n^4 + 4 = 5$, which is prime.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can expand two or three brackets and collect like terms. ○ ○ ○

I can factorise a quadratic when the x squared term is not just x squared. ○ ○ ○

I can factorise completely, using common factors and the difference of two squares. ○ ○ ○

I can find unknown constants by comparing coefficients. ○ ○ ○