



Exponential modelling

70 marks

Date _____

A Do Now

Quick questions to start.

- | | |
|--|---|
| <p>A1 GCSE Higher [2 marks]</p> <p>A town of 2400 people grows by 3% a year. Find its size after 5 years.</p> | <p>A2 Last lesson [2 marks]</p> <p>Find the gradient of $y = 5e^{-2x}$ at $x = 0$.</p> |
|--|---|

Answer

Answer

- | | |
|---|---|
| <p>A3 From chapter 12 [2 marks]</p> <p>Find the stationary point of $y = x^2 - 8x + 3$.</p> | <p>A4 From chapter 13 [2 marks]</p> <p>Find $\int_1^3 (3x^2 + 2) dx$.</p> |
|---|---|

Answer (....., ...)

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A culture holds $P = 80e^{0.12t}$ thousand bacteria t hours after it is set up. (a) Show that $\frac{dP}{dt} = kP$ and state k . (b) Find the rate of increase when $t = 5$, with units.

Hint The derivative of e^{kt} is ke^{kt} . A rate has units "quantity per unit of time".

B1 An ice sheet has area $A = 3200e^{-0.025t}$ km², t years after measuring began. Find $\frac{dA}{dt}$ when $t = 20$, and say what its sign tells you. [3 marks]

Answer $\frac{dA}{dt} = \dots\dots$

EXAMPLE 2

A club has $N = 500e^{0.08t}$ members t years after it starts. By what percentage does the membership grow each year?

Hint $N(t+1) = 500e^{0.08t} \times e^{0.08}$: each year N is multiplied by $e^{0.08}$.

B2 A dose of medicine leaves $m = 40e^{-0.35t}$ mg in the body after t hours. Find the percentage that leaves the body each hour, to 3 significant figures. [3 marks]

Answer

EXAMPLE 3

A town has $P = Ae^{0.03t}$ people t years after 2020, and $P = 15\,000$ when $t = 6$. Find (a) A , to the nearest whole number (b) the first whole number t for which $P > 20\,000$.

Hint Put $t = 6$, $P = 15\,000$ in to find A . For (b) work out P at whole numbers of years: show the values either side of 20 000.

B3 An investment is worth £ $V = Ae^{0.05t}$ after t years. After 4 years it is worth £6000. Find A , to the nearest pound, and the value after 10 years. [3 marks]

Answer $A = \dots\dots$ $V = \dots\dots$

c Practice

A-Level

C1 Sketch $P = 40 + 160e^{-0.5t}$ for $t \geq 0$, showing where it meets the P -axis and its asymptote. Say what happens to P in the long term. [4 marks]

C2 A lake holds $F = 900e^{-0.04t}$ fish t years after a disease is found. How many fish are in the lake when the population is falling at 30 fish per year? [3 marks]

Answer

C3 After t weeks of training, a trainee types $W = 70 - 50e^{-0.2t}$ words per minute.

(a) Explain what the 70 and the 50 mean in this model. [2 marks]

(b) Find the rate of improvement after 5 weeks. [2 marks]

Answer

Total for C3: 4 marks

D Exam-style questions

Edexcel exam style

D1 A scientist puts 200 bacteria in a dish of food. Two models for the number of bacteria, N , after t hours are

$$\text{Model 1: } N = 200e^{0.4t} \quad \text{Model 2: } N = 5000 - 4800e^{-0.1t}$$

(a) Find each model's prediction after 8 hours, to the nearest whole number. [3 marks]

Answer Model 1: Model 2:

(b) Show that, for Model 2, $\frac{dN}{dt} = 0.1(5000 - N)$.

[3 marks]

(c) Hence describe what each model predicts in the long term, and explain why Model 2 is more realistic for bacteria in a dish of food. [3 marks]

Total for D1: 9 marks

D2

The mass, m mg, of a drug in a patient's blood t hours after an injection is modelled by $m = 30e^{-0.25t}$. Priya says: "The mass of the drug falls by 25% every hour."

Priya is wrong.

(a) Explain why Priya is wrong, and find the correct percentage by which the mass falls each hour, to 3 significant figures. [3 marks]

Answer

- (b) The patient needs another injection once the mass falls below 10 mg. Find the first whole number of hours after which the mass is below 10 mg. [3 marks]

Answer hours

- (c) Find the rate at which the mass is decreasing when $t = 2$. [2 marks]

Answer

Total for D2: 8 marks

D3 The number of followers, F , of a new social media account t months after it starts is modelled by $F = pq^t$, where p and q are constants. After 2 months the account has 1440 followers, and after 6 months it has 7290.

- (a) Find the value of p and the value of q , and explain what each one means in this model. [6 marks]

Answer $p = \dots\dots$ $q = \dots\dots$

- (b) Hence find the first whole number of months after which the model predicts more than 50 000 followers. [3 marks]

Answer months

Total for D3: 9 marks

D4 A lake holds two kinds of fish. t years after a study begins, the number of fish of kind A is modelled by $50e^{0.2t}$, and the number of kind B by $750e^{-0.2t}$. The total number of fish is F , so $F = 50e^{0.2t} + 750e^{-0.2t}$.

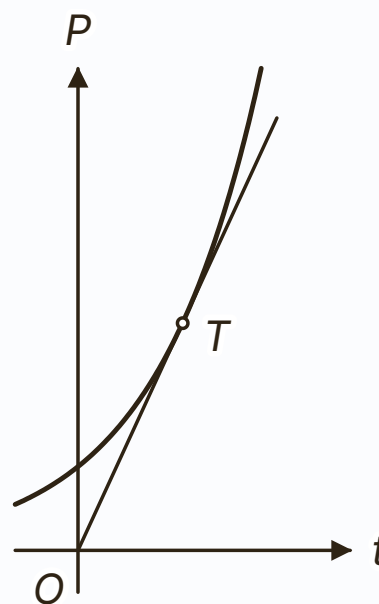
- (a) State the total number of fish when the study begins, and show that $\frac{dF}{dt} = 0$ when $e^{0.4t} = 15$. [4 marks]

- (b) Hence show that the least total number of fish is $100\sqrt{15}$, and show that this value of F is a minimum. [5 marks]

Total for D4: 9 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** A quantity is modelled by $P = Ae^{kt}$, where A and k are positive constants. A straight line through the origin O touches the graph of P against t at the point T . Show that the t -coordinate of T is $\frac{1}{k}$. Find the coordinates of T and the gradient of the line OT in terms of A , k and e . [7 marks]



Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Call the t -coordinate of T a . Write down the equation of the tangent at T , then make it pass through $(0, 0)$.
- Hint 2** The tangent at $t = a$ has gradient Ake^{ka} , so it is $P - Ae^{ka} = Ake^{ka}(t - a)$. Put $t = 0$, $P = 0$.
- Hint 3** $-Ae^{ka} = Ake^{ka}(-a)$. Divide by Ae^{ka} , which is not 0: $-1 = -ka$, so $a = \frac{1}{k}$. Then T is $\left(\frac{1}{k}, Ae\right)$ and the gradient is $Ake^{k \cdot \frac{1}{k}} = Ake$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can evaluate an exponential model and explain what its constants mean. ☐ ☐ ☐

I can find a rate of change and the percentage change per unit of time. ☐ ☐ ☐

I can find a constant in a model from data and find when a model passes a level. ☐ ☐ ☐

I can judge whether a model is realistic in the long term. ☐ ☐ ☐