# Finding areas

112 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-finding-areas-with-integration/

Date

## A Do Now

Quick questions to start.

**A1** Needed today [2 marks]

Find the exact value of

$$\int_0^1 e^{2x}\,dx.$$

Answer .....

**A2** Needed today [2 marks]

Find the exact value of

$$\int_0^{\pi/4} \sec^2 x\,dx.$$

Answer .....

**A3** Common slip [2 marks]

Find $\int_{-1}^{1} x^3\,dx$. Then find the area between $y = x^3$, the $x$-axis and the lines $x = -1$ and $x = 1$.

Answer .....

**A4** Needed today [3 marks]

Use integration by parts to find the exact value of

$$\int_0^1 xe^x\,dx.$$

Answer .....



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## B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

**EXAMPLE 1**

Find the exact area of the region bounded by the curve $y = xe^{-x}$, the $x$-axis and the line $x = 2$.

**Hint** The curve meets the $x$-axis at $x = 0$, so the limits are $0$ and $2$. Use integration by parts with $u = x$.

**B1** Find the exact area of the region bounded by the curve $y = x \sin x$ and the $x$-axis for $0 \leq x \leq \pi$. [4 marks]

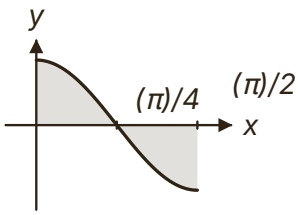
Answer .....



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**EXAMPLE 2**

Find the total area of the shaded regions between the curve $y = \cos 2x$ and the $x$-axis for $0 \leq x \leq \frac{\pi}{2}$.




**Hint** The curve crosses the axis at $x = \frac{\pi}{4}$. Integrate the two parts separately and add the sizes.

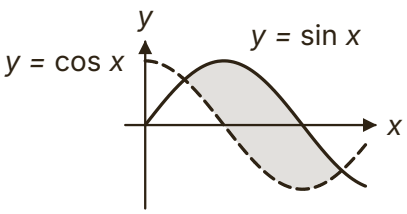
**B2** The curve $y = e^x - 2$ crosses the $x$-axis at $A$. Find the $x$-coordinate of $A$, and find the exact total area between the curve and the $x$-axis for $0 \leq x \leq 2$. [5 marks]

Answer .....


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**EXAMPLE 3**

Find the exact area of the region enclosed by $y = \sin x$ and $y = \cos x$ between their first two points of intersection for $x > 0$.




**Hint** Solve $\sin x = \cos x$ to get the limits. Then integrate (top curve − bottom curve).

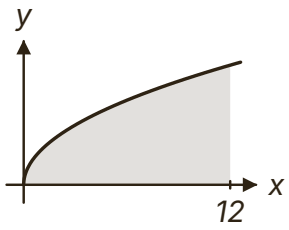
**B3** The curves $y = e^x$ and $y = 4 - 3e^{-x}$ meet at two points. Find the $x$-coordinates of the two points and the exact area of the region between the curves. [6 marks]

Answer .....


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**EXAMPLE 4**

A curve has parametric equations $x = 3t^2$, $y = 2t$, $t \geq 0$. Find the area of the region bounded by the curve, the $x$-axis and the line $x = 12$.




**Hint** Area $= \int y \frac{dx}{dt}\, dt$. Change the limits: $x = 0$ gives $t = 0$ and $x = 12$ gives $t = 2$.

**B4** A curve has parametric equations $x = \sin t$, $y = \sin 2t$, $0 \leq t \leq \frac{\pi}{2}$. Find the exact area of the region between the curve and the $x$-axis. [5 marks]

Answer .....


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**C1** Leo is asked for the area between $y = x(x-3)$ and the $x$-axis for $0 \le x \le 4$. He writes: "$\int_0^4 (x^2 - 3x)\,dx = \frac{64}{3} - 24 = -\frac{8}{3}$, so the area is $\frac{8}{3}$." Explain his mistake and find the correct area. [5 marks]

Answer .....


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**C2** A curve has parametric equations $x = 1 + t^2$, $y = 2t$, $t \geq 0$. Sam finds the area between the curve, the $x$-axis and the lines $x = 1$ and $x = 5$ by writing $\int_1^5 2t \times 2t \, dt = \frac{496}{3}$. Explain his mistake and find the correct area. [4 marks]

Answer .....

**C3** The region $R$ is bounded by the curve $y = \ln x$, the $y$-axis, the $x$-axis and the line $y = 2$. Find the exact area of $R$. [4 marks]

Answer .....


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**C4** Find the exact area of the region bounded by the curve $y = \dfrac{2}{(x+1)(x+3)}$, the coordinate axes and the line $x = 3$. Give your answer in the form $\ln k$. [5 marks]

Answer .....


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**C5** Use the substitution $u = 2x + 1$ to find the exact area of the region bounded by $y = \dfrac{x}{\sqrt{2x+1}}$, the $x$-axis and the line $x = 4$. [5 marks]

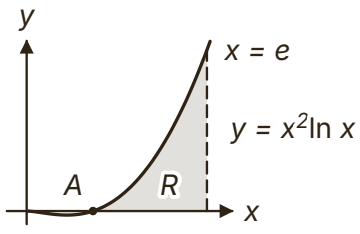
Answer .....

## D Exam-style questions

Edexcel exam style

**D1** The diagram shows part of the curve $y = x^2 \ln x$, $x > 0$. The curve meets the $x$-axis at the point $A$. The region $R$ is bounded by the curve, the $x$-axis and the line $x = e$.

**(a)** Find the coordinates of $A$. [2 marks]




Answer .....


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**(b)** Find $\int x^2 \ln x \, dx$. [4 marks]

Answer .....

**(c)** Hence find the exact area of $R$, giving your answer in the form $\frac{ae^3 + b}{9}$. [3 marks]

Answer .....

**Total for D1: 9 marks**

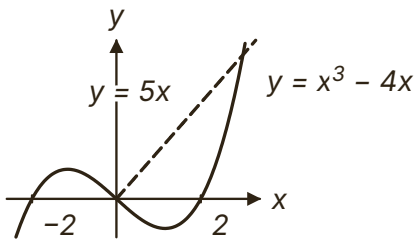

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**D2**

Priya is asked for the area of the region between the curve $y = x^3 - 4x$ and the $x$-axis. She writes: "The curve crosses the axis at $x = -2$ and $x = 2$, and $\int_{-2}^{2} (x^3 - 4x)\,dx = 0$, so the area is $0$."

Priya is wrong.

**(a)** Explain the two mistakes in her working. [2 marks]




**(b)** Find the area of the region between the curve and the $x$-axis. [4 marks]

Answer .....


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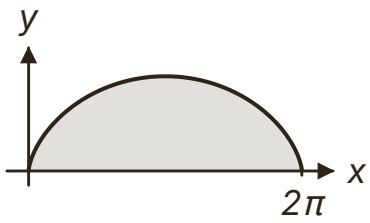
**(c)** The line $y = 5x$ meets the curve at the origin and at a point $P$ with $x > 0$. Find the area of the region bounded by the line and the curve between $O$ and $P$. [4 marks]

Answer .....

**Total for D2: 10 marks**


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**D3** The curve $C$ has parametric equations $x = t - \sin t$, $y = 1 - \cos t$, $0 \leq t \leq 2\pi$. The region $S$ is bounded by $C$ and the $x$-axis. Show that the area of $S$ is $3\pi$. [8 marks]



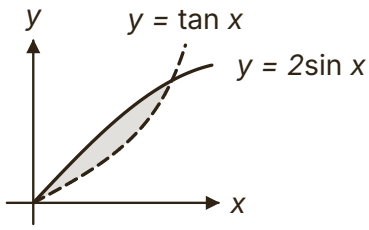


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**D4** The diagram shows the curves $y = 2\sin x$ and $y = \tan x$ for $0 \leq x < \frac{\pi}{2}$.

The shaded region lies between the curves.

**(a)** Show that the curves meet where $x = 0$ and where $x = \frac{\pi}{3}$. [3 marks]




**(b)** Show that $\int \tan x \, dx = -\ln|\cos x| + c$. [2 marks]


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**(c)** Hence find the exact area of the shaded region, in the form $a + b \ln 2$. [4 marks]

Answer .....

**Total for D4: 9 marks**

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**D5** $f(x) = \dfrac{x + 7}{(x + 1)(x + 3)}$, $x \geq 0$.

**(a)** Express $f(x)$ in partial fractions. [3 marks]

Answer .....


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**(b)** Hence find the exact area of the region bounded by $y = f(x)$, the coordinate axes and the line $x = 3$, giving your answer in the form $k \ln 2$. [4 marks]

Answer .....

**Total for D5: 7 marks**

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**D6** The curve $y = \dfrac{6x}{x^2 + 1}$ and the line $y = 3x$ enclose a region $T$ for $x \geq 0$.

**(a)** Show that the curve and the line meet where $x = 0$ and where $x = 1$. [2 marks]


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**(b)** Find the exact area of $T$. [5 marks]

Answer .....

**Total for D6: 7 marks**


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## E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

**E1** For $x \geq 0$ the curve $y = e^{-x}\sin x$ crosses the $x$-axis at $x = 0, \pi, 2\pi, \ldots$. Let $A_n$ be the area of the region between the curve and the $x$-axis for $(n-1)\pi \leq x \leq n\pi$. Show that $A_{n+1} = e^{-\pi}A_n$, and find the exact total area of all these regions. [10 marks]



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## Extension hints

Cover these with paper. Uncover one at a time.

**E1**

**Hint 1** Use integration by parts twice on $\int e^{-x} \sin x \, dx$, then rearrange to find it.

**Hint 2** At $x = n\pi$ the sine is $0$ and the cosine is $(-1)^n$. Work out $A_n$ in general, taking the size of each integral.

**Hint 3** The areas form a geometric series. Add them all with the sum to infinity.

## How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the area between a curve and the x-axis, splitting the region where the curve crosses the axis. ○ ○ ○

I can find the area between two curves, or a curve and a line, from their points of intersection. ○ ○ ○

I can find the area under a parametric curve using the integral of y times dx/dt, with limits in t. ○ ○ ○

I can choose substitution, parts or partial fractions to do the integration an area needs. ○ ○ ○



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