

Functions and their roots

71 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-functions-and-their-roots/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] Solve $5x^2 + 9x - 2 = 0$.
- A2** Last lesson [2 marks] Write $x^2 - 16x + 71$ in the form $(x + p)^2 + q$.

Answer $x = \dots\dots$

Answer

- A3** From chapter 1 [2 marks] Write 25^x as a power of 5.
- A4** GCSE Higher [2 marks] Solve $(2x - 1)^2 = 49$.

Answer

Answer $x = \dots\dots$

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$f(x) = x^2 - 6x$, $g(x) = \sqrt{2x + 1}$. Find
(a) $f(-3) + g(4)$ (b) $f(\sqrt{3})$ (c) both values of a with $f(a) = 16$.

Hint Put the number in place of x everywhere. In (c) write $a^2 - 6a = 16$ and make one side 0.

B1 $f(x) = 2x^2 + 5x$, [4 marks]
 $g(x) = \sqrt{5x - 1}$. Find
(a) $f(-2) + g(2)$ (b) $f(\sqrt{2})$ (c) both values of a with $f(a) = 3$.

Answer (a) ... (b) ... (c) ...

EXAMPLE 2

(a) Find the roots of $h(x) = x^3 - 2x^2 - 15x$. (b) Solve $f(x) = g(x)$, where $f(x) = x^2 + 4x - 9$ and $g(x) = 7x + 9$.

Hint A root is a value of x that makes the function 0. In (a) take out x first. In (b) put $f(x)$ equal to $g(x)$.

B2 (a) Find the roots of $k(x) = 3x^3 - 5x^2 - 2x$. [4 marks]
(b) Solve $p(x) = q(x)$, where $p(x) = x^2 - 3x + 1$ and $q(x) = 2x + 37$.

Answer (a) (b) $x = \dots\dots$

EXAMPLE 3

$f(x) = x^2 + 12x + 33$. (a) Write $f(x)$ as $(x + p)^2 + q$. (b) Find its roots in surd form. (c) State its minimum value and where it occurs.

Hint Halve the 12. A square is never negative, so the smallest it can be is 0.

B3 $g(x) = x^2 + 18x + 70$. [5 marks]

(a) Write $g(x)$ as $(x + p)^2 + q$. (b) Find its roots in surd form. (c) State its minimum value and where it occurs.

Answer (a) ... (b) ... (c) ...

EXAMPLE 4

Find all the real roots of (a)

$$f(x) = x^6 - 26x^3 - 27 \quad (\text{b})$$

$$g(x) = 3x - 7\sqrt{x} - 6.$$

Hint (a) Let $u = x^3$, so $x^6 = u^2$. (b) Let $u = \sqrt{x}$, so $x = u^2$. A square root cannot be negative.

B4 Find all the real roots of [5 marks]

(a)

$$j(x) = x^6 + 19x^3 - 216$$

(b)

$$k(x) = x^{\frac{2}{3}} + 3x^{\frac{1}{3}} - 4.$$

Answer (a) (b)

- C1** The functions f and g are given by $f(x) = 3x^3 + 5x$ and $g(x) = 16x^2$, [3 marks]
 $x \in \mathbb{R}$. Find the three values of x for which $f(x) = g(x)$.

Answer $x = \dots\dots$

- C2** $f(x) = 3x^2 - 12x + 17$. Write $f(x)$ in the form $a(x + p)^2 + q$. Hence [3 marks]
explain why $f(x) > 0$ for every value of x .

Answer $f(x) = \dots\dots$

- C3** Solve $5^{2x+1} - 26(5^x) + 5 = 0$. [4 marks]

Answer $x = \dots\dots$

D1 Sam solves $x - 3\sqrt{x} - 28 = 0$. He writes: let $u = \sqrt{x}$, so $u^2 - 3u - 28 = 0$, $(u - 7)(u + 4) = 0$, $u = 7$ or $u = -4$, so $x = 49$ or $x = 16$.

Sam's answer is not correct.

(a) Explain Sam's mistake, and give the correct solution. [2 marks]

Answer $x = \dots\dots$

(b) Solve $2x + 3\sqrt{x} - 20 = 0$. [3 marks]

Answer $x = \dots\dots$

Total for D1: 5 marks

D2 The function f is defined by $f(x) = x^2 - 12x + 1$, $x \in \mathbb{R}$.

(a) Write $f(x)$ in the form $(x + p)^2 + q$, where p and q are integers. [2 marks]

Answer

- (b) Hence write down the minimum value of $f(x)$, and the value of x at which it occurs. [2 marks]

Answer minimum at $x = \dots\dots$

- (c) Hence find the roots of $f(x)$, giving your answers in surd form. [2 marks]

Answer $x = \dots\dots$

- (d) Hence find the four roots of $g(x) = x^4 - 12x^2 + 1$, giving exact answers. [3 marks]

Answer $x = \dots\dots$

Total for D2: 9 marks

D3 The function f is defined by $f(x) = 4^x - 12(2^x) + 32$, $x \in \mathbb{R}$.

- (a) Write $f(x)$ in the form $(2^x - a)(2^x - b)$, where a and b are integers. [2 marks]

Answer

(b) Hence find the two roots of $f(x)$.

[2 marks]

Answer $x = \dots\dots$

(c) Show that $f(x) = (2^x - 6)^2 - 4$, and hence write down the least value of $f(x)$. [3 marks]

Answer least value $\dots\dots$

(d) Explain why the equation $f(x) = -5$ has no solutions. [1 mark]

Total for D3: 8 marks

D4 The function f is defined by $f(x) = 3x^2 - 8x$, $x \in \mathbb{R}$.

(a) Find $f(-2)$. [1 mark]

Answer

(b) Given that $f(p) = 3$, find the two possible values of p . [3 marks]

Answer $p = \dots\dots$

- (c) Find $f(2)$. Hence find the value of q , other than 2, for which $f(q) = f(2)$.

[3 marks]

Answer $q = \dots\dots$

Total for D4: 7 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Find all four roots of $f(x) = (x^2 + 2x)^2 - 11(x^2 + 2x) + 24$.

[6 marks]

Answer $x = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** The same bracket appears twice. Call it u .
Hint 2 With $u = x^2 + 2x$, the equation is $u^2 - 11u + 24 = 0$, so $u = 3$ or $u = 8$.
Hint 3 Now solve $x^2 + 2x = 3$ and $x^2 + 2x = 8$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can work out values of a function and find inputs that give an output.

☐ ☐ ☐

I can find the roots of a function and solve $f(x) = g(x)$.

☐ ☐ ☐

I can use a completed square to find roots and a least value.

☐ ☐ ☐

I can solve an equation that is a quadratic in disguise.

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