Functions, mappings and composite functions

119 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-functions-mappings-and-composite-functions/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Given $f(x) = 3x^2 - 2$, find $f(-2)$ and $f\left(\frac{1}{3}\right)$.

Answer .....

A2 Needed today [3 marks]

Write $x^2 - 6x + 11$ in the form $(x - p)^2 + q$. Hence state the least value of $x^2 - 6x + 11$.

Answer .....

A3 From chapter 1 [2 marks]

Solve $2x^2 - 5x - 3 = 0$.

Answer .....

A4 Needed today [2 marks]

Given $g(x) = 2x + 1$, write $g(x + 3)$ in its simplest form.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

The function f is defined by $f(x) = x^2 - 4x + 1$, $x \in \mathbb{R}$. Find the range of f.

Hint Complete the square. The least value of a squared bracket is 0.

B1 The function f is defined by $f(x) = x^2 + 6x + 4$, $x \in \mathbb{R}$. Find the range of f. [3 marks]

Answer .....

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EXAMPLE 2

$f(x) = 2x - 1$ and $g(x) = x^2 + 3$, $x \in \mathbb{R}$. Find

(a) $fg(2)$

(b) $gf(x)$ in its simplest form.

Hint In $fg(x)$ the function written next to x is applied first: $fg(x) = f(g(x))$.

B2 $f(x) = 3x + 2$ and $g(x) = x^2 - 1$, $x \in \mathbb{R}$. Find [4 marks]

(a) $fg(3)$

(b) $gf(x)$ in its simplest form.

Answer .....

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EXAMPLE 3

$f(x) = 4x - 3$, $x \in \mathbb{R}$, and $g(x) = \dfrac{1}{x+2}$, $x > -2$. Solve $fg(x) = 5$.

Hint Write $fg(x)$ as one expression, then solve. Check that your answer is in the domain of g.

B3 $f(x) = 2x + 5$, $x \in \mathbb{R}$ [4 marks]

, and $g(x) = \dfrac{1}{x-1}$,

$x > 1$. Solve

$fg(x) = 9$.

Answer .....

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EXAMPLE 4

$f(x) = x - 5$, $x \in \mathbb{R}$, and $g(x) = \sqrt{x}$, $x \geq 0$. Find the largest possible domain of gf.

Hint g can only take inputs that are 0 or more. So the output of f must be 0 or more.

B4 $f(x) = 4 - x^2$, $x \in \mathbb{R}$, and $g(x) = \sqrt{x}$, $x \geq 0$. [4 marks]

(a) Explain why $gf(5)$ is not defined.

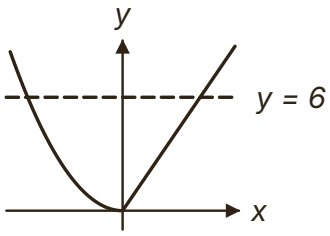
(b) Find the largest possible domain of gf.

Answer .....

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EXAMPLE 5

$f(x) = \begin{cases} x^2, & x < 0 \\ 3x, & x \geq 0 \end{cases}$. Sketch $y = f(x)$ and solve $f(x) = 6$.



Hint Solve in each piece, then keep only the answers that lie in that piece's interval.

B5 $f(x) = \begin{cases} 2x + 1, & x \leq 1 \\ 5 - 2x, & x > 1 \end{cases}$. Solve $f(x) = 2$. [4 marks]

Answer .....

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C1 For each mapping, state whether it is a function. If it is, state whether it is one-to-one or many-to-one. Give a reason each time. [5 marks]

(a) $y = \dfrac{1}{x-2}$, $x \in \mathbb{R}$

(b) $y = x^2$, $x \in \mathbb{R}$

(c) $y^2 = x$, $x \geq 0$

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C2 $f(x) = x + 4$ and $g(x) = 3x^2$, $x \in \mathbb{R}$. Lily works out $fg(1)$. She writes: "$f(1) = 5$, then $g(5) = 75$, so $fg(1) = 75$." Explain her mistake and find the correct value of $fg(1)$. [4 marks]

Answer .....

C3 $f(x) = 2x - 3$ and $g(x) = x^2$, $x \in \mathbb{R}$. Solve $fg(x) = gf(x)$, giving your answers in surd form. [5 marks]

Answer .....

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C4 $f(x) = x^2 + 2$, $x \in \mathbb{R}$, and $g(x) = \dfrac{6}{x}$, $x > 0$. Find $gf(x)$ and the range of gf. [4 marks]

Answer .....

C5 $h(x) = x^2 - 4x + 7$, $0 \leq x \leq 5$. Ben says the range of h is $7 \leq h(x) \leq 12$, because $h(0) = 7$ and $h(5) = 12$. Explain why Ben is wrong and find the correct range. [5 marks]

Answer .....

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C6 $f(x) = \dfrac{3}{x-1}$, $x \in \mathbb{R}$, $x \neq 1$. [6 marks]

(a) Show that $ff(x) = \dfrac{3(x-1)}{4-x}$.

(b) Hence solve $ff(x) = x$, giving your answers in surd form.

Answer .....

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D1 The functions f and g are defined by $f(x) = x^2 - 6x + 5$, $x \in \mathbb{R}$, and $g(x) = 2x + 1$, $x \in \mathbb{R}$.

(a) Find the range of f. [3 marks]

Answer .....

(b) Show that $fg(x) = 4x^2 - 8x$. [2 marks]

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(c) Solve $fg(x) = 12$. [3 marks]

Answer .....

(d) Find the range of fg. [2 marks]

Answer .....

Total for D1: 10 marks

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D2

$f(x) = x^2 + 1$, $x \in \mathbb{R}$, and $g(x) = \sqrt{x - 3}$, $x \geq 3$. Ana says: "g has domain $x \geq 3$, so the domain of gf is also $x \geq 3$."

Ana is wrong.

(a) Explain the mistake Ana makes. [2 marks]

(b) Find the largest possible domain of gf. [4 marks]

Answer .....

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(c) Solve $gf(x) = \sqrt{6}$. [3 marks]

Answer .....

Total for D2: 9 marks

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D3 The function f is defined by $f(x) = x^2 - 2x - 3$, $x \in \mathbb{R}$. Find, in exact form, every solution of the equation $ff(x) = 3$. [7 marks]

Answer .....

D4 The function f is defined by $f(x) = \begin{cases} x + 4, & x < -1 \\ x^2 + 2, & x \geq -1 \end{cases}$

(a) Find the value of $f(-3) + f(2)$. [2 marks]

Answer .....

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(b) Solve $f(x) = 11$. [3 marks]

Answer .....

(c) Find $ff(-2)$. [2 marks]

Answer .....

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(d) Find the set of values of k for which the equation $f(x) = k$ has exactly three solutions. [4 marks]

Answer .....

Total for D4: 11 marks

D5 $f(x) = 2x + 3$, $x \in \mathbb{R}$, and $g(x) = \dfrac{12}{x+1}$, $x > -1$.

(a) Find $gf(x)$ in its simplest form, and state its domain. [3 marks]

Answer .....

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(b) Solve $gf(x) = f(x)$. [4 marks]

Answer .....

(c) State the range of gf. [2 marks]

Answer .....

Total for D5: 9 marks

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D6 $f(x) = 2x^2 - 8x + 5, -1 \leq x \leq 5.$

(a) Find the range of f. [4 marks]

Answer .....

(b) Find the set of values of k for which $f(x) = k$ has two distinct solutions. [3 marks]

Answer .....

Total for D6: 7 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The function f is defined by $f(x) = x^2 - 4x$, $x \in \mathbb{R}$. Find the set of values of k for which the equation $ff(x) = k$ has exactly four real solutions. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Let $y = f(x)$. First solve $f(y) = k$ for y, then ask how many x give each value of y.

Hint 2 $f(y) = k$ gives $y = 2 \pm \sqrt{k+4}$. The range of f is $f(x) \geq -4$, and $f(x) = y$ has two solutions only when $y > -4$.

Hint 3 $y = 2 + \sqrt{k+4}$ is always more than -4. Find when $y = 2 - \sqrt{k+4}$ is more than -4, and when the two values of y are different.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can say whether a mapping is a function, and whether it is one-to-one or many-to-one.	○ ○ ○
I can find the range of a function on a given domain, using its turning point.	○ ○ ○
I can work out fg(x) and gf(x), applying g first in fg(x).	○ ○ ○
I can find the domain and range of a composite function and solve equations involving it.	○ ○ ○

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