

Geometric problems and modelling with vectors

70 marks



Hints online, answers free with an account:
mathswariz.uk/worksheets/a-level-geometric-problems-and-modelling/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] **A2** Last lesson [2 marks]
 A is $(2, -1)$ and B is $(14, 8)$. Find the coordinates of the point P on AB with $AP : PB = 1 : 2$.
Find a unit vector in the direction of $20\mathbf{i} - 48\mathbf{j}$.

Answer

Answer (.....,.....)

- A3** GCSE Higher [2 marks] **A4** From chapter 3 [2 marks]
A ship sails 6 km due east and then 8 km due south. Find the bearing of its finishing point from its start, to the nearest degree.
Solve $2\lambda = 1 - \mu$ and $\lambda = 3\mu$.

Answer $\lambda = \dots\dots$ $\mu = \dots\dots$

Answer°

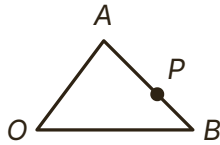
B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$, P is on AB with $AP : PB = 3 : 2$. (a) Find $\vec{OP}$. (b) $\vec{OX} = 2\mathbf{a} + 3\mathbf{b}$. Show that O, P, X are collinear.



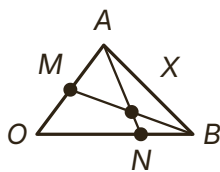
Hint $\vec{OP} = \vec{OA} + \frac{3}{5}\vec{AB}$. Collinear: parallel, with a point in common.

B1 $\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$, P is on AB with $AP : PB = 1 : 3$. (a) Find $\vec{OP}$. (b) $\vec{OY} = 6\mathbf{a} + 2\mathbf{b}$. Show that O, P, Y are collinear. [4 marks]

Answer (a) $\vec{OP} = \dots\dots$

EXAMPLE 2

$\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$, M is the midpoint of OA and $ON : NB = 2 : 1$. AN and BM meet at X . Find $\vec{OX}$ and the ratio $AX : XN$.



Hint Reach X along AN and along BM , each with its own unknown. Compare coefficients of $\mathbf{a}$ and $\mathbf{b}$.

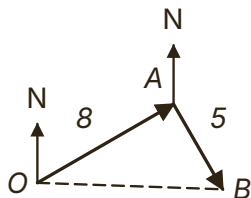
B2 In triangle OAB , $\vec{OA} = \mathbf{a}$, $\vec{OB} = \mathbf{b}$, $OP : PA = 1 : 2$ and Q is the midpoint of OB . AQ and BP meet at X . Find $\vec{OX}$. [5 marks]

Answer $\vec{OX} = \dots\dots$

EXAMPLE 3

A walker goes 8 km on a bearing of 060° to A , then 5 km on 150° to B .

With $\mathbf{i}$ east, $\mathbf{j}$ north, find $\overrightarrow{OB}$, OB and the bearing of B from O .



Hint A leg d on bearing θ is $d \sin \theta \mathbf{i} + d \cos \theta \mathbf{j}$.

B3 A ship sails 12 km on a bearing of 040° to A , then 7 km on 130° to B . Find $\overrightarrow{OB}$ (3 s.f.), OB , and the bearing of B from O (1 d.p.). [5 marks]

Answer $OB = \dots\dots$ km $\dots\dots^\circ$

c Practice

A-Level

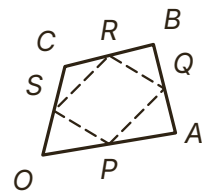
C1 A buoy drifts at constant velocity. At 10:00 it is at $(-3\mathbf{i} + 5\mathbf{j})$ km and at 10:30 at $(4\mathbf{i} - 19\mathbf{j})$ km ($\mathbf{i}$ east, $\mathbf{j}$ north). Find its velocity, its speed and its bearing of drift, to 1 d.p. [4 marks]

Answer velocity $\dots\dots$ speed $\dots\dots$ bearing $\dots\dots^\circ$

- C2** A 0.4 kg particle has velocity $(3\mathbf{i} - 2\mathbf{j}) \text{ m s}^{-1}$ at $t = 0$ and $(11\mathbf{i} + 4\mathbf{j}) \text{ m s}^{-1}$ at $t = 2 \text{ s}$. Using $\mathbf{a} = \frac{\mathbf{v} - \mathbf{u}}{t}$ and $\mathbf{F} = m\mathbf{a}$, find $\mathbf{a}$, $|\mathbf{F}|$, and the angle between $\mathbf{F}$ and $\mathbf{i}$ to 1 d.p. [4 marks]

Answer $\mathbf{a} = \dots\dots$ $|\mathbf{F}| = \dots\dots \text{ N}$ angle $\dots\dots^\circ$

- C3** $OABC$ is a quadrilateral with $\overrightarrow{OA} = \mathbf{a}$, $\overrightarrow{OB} = \mathbf{b}$ and $\overrightarrow{OC} = \mathbf{c}$. P, Q, R and S are the midpoints of OA, AB, BC and CO . Prove that $PQRS$ is a parallelogram. [4 marks]



D Exam-style questions

Edexcel exam style

- D1** Two forces $\mathbf{F}_1 = (2\mathbf{i} - 5\mathbf{j}) \text{ N}$ and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j}) \text{ N}$ act on a particle. Their resultant $\mathbf{R}$ acts in the direction of the vector $3\mathbf{i} + \mathbf{j}$.

- (a) Find the angle between $\mathbf{R}$ and $\mathbf{i}$, to 1 d.p. [2 marks]

Answer $\dots\dots^\circ$

(b) Show that $p - 3q = -17$.

[3 marks]

(c) Given that $q = 7$, hence find p and the magnitude of $\mathbf{R}$, in simplified surd form.

[3 marks]

Answer $p = \dots\dots$ $|\mathbf{R}| = \dots\dots$ N

Total for D1: 8 marks

D2

$\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. M is on OA with $OM : MA = 2 : 1$, N is the midpoint of AB , and C is on OB extended with $\overrightarrow{OC} = 3\mathbf{b}$. Priya says that M , N and C lie on a straight line.

Priya is wrong.

(a) Find $\overrightarrow{MN}$ and $\overrightarrow{MC}$, and show that M , N and C are not collinear. [3 marks]

- (b) The point D on OB extended has $\overrightarrow{OD} = k\mathbf{b}$, and M , N and D are collinear. Hence find k and the ratio $MN : ND$. [3 marks]

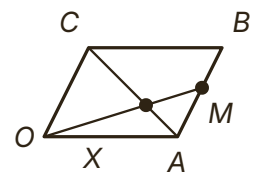
Answer $k = \dots\dots$ $MN : ND = \dots\dots$

Total for D2: 6 marks

- D3** $OABC$ is a parallelogram with $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OC} = \mathbf{c}$. M is the point on AB with $AM : MB = 1 : 2$. The line OM meets the diagonal AC at X .

- (a) Find $\overrightarrow{OM}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.

[2 marks]



Not drawn accurately

Answer $\overrightarrow{OM} = \dots\dots$

- (b) By writing $\overrightarrow{OX}$ in two ways, find the ratio $OX : XM$.

[4 marks]

Answer $OX : XM = \dots\dots$

(c) Hence find the ratio $AX : XC$.

[2 marks]

Answer $AX : XC = \dots\dots$

Total for D3: 8 marks

D4 At 12:00 ship P is at $(2\mathbf{i} + 10\mathbf{j})$ km with constant velocity $(3\mathbf{i} - 4\mathbf{j})$ km h⁻¹, and ship Q is at $(-5\mathbf{i} + 3\mathbf{j})$ km with constant velocity $(6\mathbf{i} + 3\mathbf{j})$ km h⁻¹. $\mathbf{i}$ points east and $\mathbf{j}$ north.

(a) Find the speed of Q , to 3 s.f.

[2 marks]

Answer $\dots\dots$ km h⁻¹

(b) Show that, t hours after 12:00, $\overrightarrow{PQ} = (3t - 7)\mathbf{i} + (7t - 7)\mathbf{j}$.

[3 marks]

(c) Hence find the time at which Q is due north of P , and the distance between the ships then.

[3 marks]

Answer time $\dots\dots$ distance $\dots\dots$ km

Total for D4: 8 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** In triangle OAB , $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. A median joins a vertex to the midpoint of the opposite side. Prove that the three medians meet at one point G , and find $\overrightarrow{OG}$. [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Write the median from A and the median from B with their own unknowns, as in Example 2.
- Hint 2** Compare coefficients to find where these two medians meet.
- Hint 3** Then show that point lies on the median from O , to the midpoint of AB .

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the position vector of a point dividing a line in a ratio. ☐ ☐ ☐

I can prove points are collinear. ☐ ☐ ☐

I can find where two lines meet by comparing coefficients. ☐ ☐ ☐

I can model journeys, velocities and forces with vectors. ☐ ☐ ☐