Geometric sequences and series

113 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-geometric-sequences-and-series/

Date

A Do Now

Quick questions to start. They use what you need today.

A1 Needed today [2 marks]

Solve $3 \times 2^x = 600$, giving x to 3 significant figures.

Answer .....

A2 Common slip [3 marks]

Find the smallest integer n for which $0.8^n < 0.05$.

Answer .....

A3 Needed today [2 marks]

A geometric sequence has first term 6 and common ratio $-\frac{1}{2}$. Find the 4th term.

Answer .....

A4 Needed today [2 marks]

In a geometric sequence $u_2 = 12$ and $u_5 = -96$. Find the common ratio.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

In a geometric sequence $u_3 = 18$ and $u_6 = 486$. Find the first term, the common ratio and u_{10}.

Hint Divide the two terms: $\dfrac{ar^5}{ar^2} = r^3$.

B1 In a geometric sequence $u_2 = 40$ and $u_5 = \dfrac{5}{8}$. Find the first term, the common ratio and u_8 as an exact fraction. [4 marks]

Answer .....

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EXAMPLE 2

In a geometric series $u_3 = 36$ and $u_5 = 81$. Find the two possible values of S_6.

Hint $r^2 = \dfrac{81}{36}$ has two square roots. Work out a and S_6 for each one.

B2 In a geometric series $u_2 = -12$ and $u_4 = -3$. Find the two possible values of S_5. [5 marks]

Answer .....

EXAMPLE 3

Prove that the sum of the first n terms of a geometric series with first term a and common ratio r, $r \neq 1$, is $S_n = \dfrac{a(1 - r^n)}{1 - r}$.

Hint Write S_n and rS_n underneath it. Subtract: almost every term cancels.

B3 Prove that $1 + 2 + 4 + \cdots + 2^{n-1} = 2^n - 1$. [4 marks]

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EXAMPLE 4

Find the least number of terms of $4 + 6 + 9 + \cdots$ for which the sum is greater than 1000.

Hint Write S_n with $r = 1.5$, make 1.5^n the subject, then take logs.

B4 [5 marks]

The geometric sequence $200, 180, 162, \ldots$ has nth term u_n. Find the first term that is less than 20, and its value.

Answer .....

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EXAMPLE 5

The sum of the first n terms of $3 + 6 + 12 + \cdots$ is 3069. Find n.

Hint Set the sum formula equal to 3069 and make 2^n the subject.

B5 The sum of the first n terms of $5 - 10 + 20 - 40 + \cdots$ is -1705. Find n. [5 marks]

Answer .....

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C1 A student wants the least n for which $0.75^n < 0.01$. She writes that $n \log 0.75 < \log 0.01$, so $n < \dfrac{\log 0.01}{\log 0.75} = 16.01$, and so $n = 16$. Explain her mistake and find the correct value of n. [4 marks]

Answer $n = \ldots\ldots$

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C2 The expressions $(2k + 3)$, $(k + 6)$ and k are consecutive terms of a geometric sequence. Find the two possible values of k and the common ratio in each case. [5 marks]

Answer .....

C3 Find the sum of the first 8 terms of $2 - 6 + 18 - 54 + \cdots$. [4 marks]

Answer .....

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C4 For the series $3 + 6 + 12 + \cdots$, a student finds $u_5 + u_6 + \cdots + u_{12}$ by working out $S_{12} - S_5$. Explain why this is wrong, and find the correct value. [4 marks]

Answer .....

C5 A geometric series has first term 5, and the sum of its first 3 terms is 65. Find the possible values of the common ratio. [4 marks]

Answer .....

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D1 A geometric series has second term 50 and fifth term 25.6.

(a) Find the common ratio and the first term. [3 marks]

Answer .....

(b) Find the sum of the first 10 terms, to 1 decimal place. [2 marks]

Answer .....

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(c) Find the least value of n for which $S_n > 300$. [4 marks]

Answer .....

Total for D1: 9 marks

D2

> A geometric sequence has $u_4 = 16$ and $u_6 = 64$. Kai writes: "$r^2 = 4$, so $r = 2$ and $a = 2$. There is only one sequence."

Kai is wrong.

(a) Explain Kai's mistake, and give every possible pair of values of r and a. [3 marks]

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(b) Find S_8 for each pair. [3 marks]

Answer .....

(c) For the series in which every term is positive, find the least n for which $S_n > 10^6$. [3 marks]

Answer .....

Total for D2: 9 marks

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D3 A geometric series has positive terms. The sum of the first 2 terms is 16 and the sum of the first 4 terms is 80. Find the sum of the first 10 terms. [7 marks]

Answer .....

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D4 The first three terms of a geometric sequence are $(x + 4)$, x and $(x - 3)$.

(a) Find the value of x. [3 marks]

Answer .....

(b) Find the 10th term as an exact fraction. [3 marks]

Answer .....

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(c) Find the least n for which the sum of the first n terms is greater than 63. [4 marks]

Answer .....

Total for D4: 10 marks

D5 A geometric series has first term a and common ratio r, where $r \neq 1$.

(a) Prove that the sum of the first n terms is $\dfrac{a(1-r^n)}{1-r}$. [4 marks]

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(b) Hence show that $1 + 3 + 9 + \cdots + 3^{n-1} = \dfrac{3^n - 1}{2}$, and find n when this sum is 9841. [5 marks]

Answer .....

Total for D5: 9 marks

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D6 The sum of the first n terms of a series is $S_n = 2(3^n - 1)$. Prove that the series is geometric, and find $u_{11} + u_{12} + \cdots + u_{20}$ in the form $k \times 3^m(3^m - 1)$, where k and m are integers. [7 marks]

Answer $k =$ ..... , $m =$ .....

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The numbers a, b and c are three different consecutive terms of a geometric sequence. The numbers a, $2b$ and $3c$ are consecutive terms of an arithmetic sequence. Given that $a + b + c = 26$, find a, b and c. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Let $b = ar$ and $c = ar^2$.

Hint 2 In an arithmetic sequence the middle term is the mean of the other two, so $2(2b) = a + 3c$.

Hint 3 You should get $3r^2 - 4r + 1 = 0$. One root gives three equal numbers.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the first term and common ratio from two terms, including a negative ratio.	○ ○ ○
I can prove and use the formula for the sum of a geometric series.	○ ○ ○
I can use logarithms to find the least n for a condition, turning the inequality round when I divide by a negative log.	○ ○ ○
I can solve problems where three expressions are consecutive terms.	○ ○ ○

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