

Gradients, tangents and normals

73 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-gradients-tangents-and-normals/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks]
Find an equation of the line through $(-2, 5)$ with gradient $-\frac{3}{4}$, in the form $ax + by + c = 0$ where a , b and c are integers.
- A2** Needed today [2 marks]
The line L has equation $2x - 5y + 3 = 0$. Find the gradient of a line perpendicular to L .

Answer

Answer

- A3** Last lesson [2 marks]
Given that $y = 4x^3 - \frac{6}{x^2}$,
find $\frac{dy}{dx}$.
- A4** From chapter 2 [2 marks]
Solve $2x^2 - 3x - 9 = 0$.

Answer $x = \dots\dots$ or $x = \dots\dots$

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find an equation of the tangent to the curve $y = x^3 - 4x^2 + 2x + 7$ at the point $(3, 4)$.

Hint Differentiate, then put $x = 3$ into $\frac{dy}{dx}$ to get the gradient. Use $y - y_1 = m(x - x_1)$.

B1 Find an equation of the [3 marks]

tangent to the curve
 $y = 2x^3 + x^2 - 5x + 1$
at the point where
 $x = -1$.

Answer $y = \dots\dots$

EXAMPLE 2

Find an equation of the normal to the curve $y = \frac{x^2 + 12}{x}$ at the point where $x = 2$. Give your answer in the form $ax + by + c = 0$, where a , b and c are integers.

Hint Split the fraction: $y = x + 12x^{-1}$. The normal is perpendicular to the tangent, so its gradient is $-\frac{1}{m}$.

B2 Find an equation of the [4 marks]

normal to the curve
 $y = \frac{4}{x^2} + 3x$ at the
point where $x = 2$. Give
your answer in the form
 $ax + by + c = 0$, where
 a , b and c are integers.

Answer

EXAMPLE 3

Find the coordinates of the points on the curve $y = 2x^3 - 3x^2 - 10x + 1$ at which the tangent is parallel to the line $y = 2x$.

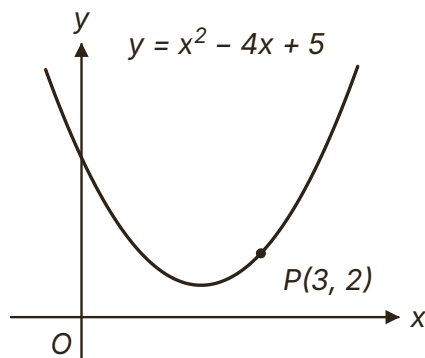
Hint Parallel lines have equal gradients. Set $\frac{dy}{dx} = 2$ and solve.

B3 Find the coordinates of the points on the curve $y = x^3 - 6x^2 + 4x + 3$ at which the tangent is perpendicular to the line $x - 5y = 2$. [4 marks]

Answer (....., ...) and (....., ...)

EXAMPLE 4

The point $P(3, 2)$ lies on the curve $y = x^2 - 4x + 5$. The normal to the curve at P meets the curve again at Q . Find the coordinates of Q .



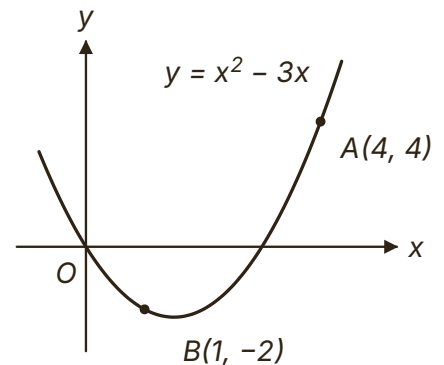
Not drawn accurately

Hint Find the normal at P , then solve it with the curve. One root of the quadratic is $x = 3$, which helps you factorise.

B4 The point $P(1, 0)$ lies on the curve $y = x^3 - 2x^2 - 3x + 4$. The tangent to the curve at P meets the curve again at Q . Find the coordinates of Q . [5 marks]

Answer (....., ...)

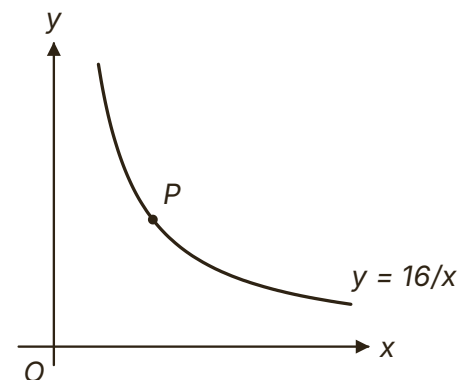
- C1** The points $A(4, 4)$ and $B(1, -2)$ lie on the curve $y = x^2 - 3x$. The tangent to the curve at A meets the normal to the curve at B at the point R . Find the coordinates of R . [5 marks]



Not drawn accurately

Answer (....., ...)

- C2** The point P on the curve $y = \frac{16}{x}$ has x -coordinate 2. The tangent to the curve at P meets the x -axis at A and the y -axis at B . Find the area of triangle OAB , where O is the origin. [4 marks]



Answer

Not drawn accurately

- C3** Find the coordinates of the point on the curve $y = 6\sqrt{x}$ at which the normal to the curve has gradient -2 . [3 marks]

Answer (....., ...)

D1 The curve C has equation $y = x^3 - 3x^2 - 4x + 7$. The point P on C has x -coordinate -1 .

- (a) The tangent to C at the point Q is parallel to the tangent to C at P . Find the coordinates of Q . [4 marks]

Answer (....., ...)

- (b) Find an equation of the tangent to C at P , in the form $y = mx + c$. [3 marks]

Answer $y = \dots\dots$

- (c) The tangent to C at P meets C again at the point T . Find the coordinates of T . Solutions relying entirely on calculator technology are not acceptable. [4 marks]

Answer (....., ...)

Total for D1: 11 marks

D2

Mia is finding the normal to the curve $y = 2x^3 - 3x + 5$ at the point where $x = 1$. She writes: "At $x = 1$, $y = 4$, so the gradient of the curve is 4. The normal has gradient -4 , so the normal is $y - 4 = -4(x - 1)$."

Mia's answer is wrong.

(a) Describe the two mistakes Mia has made. [2 marks]

(b) Find an equation of the normal to the curve at the point where $x = 1$, in the form $ax + by + c = 0$, where a , b and c are integers. [3 marks]

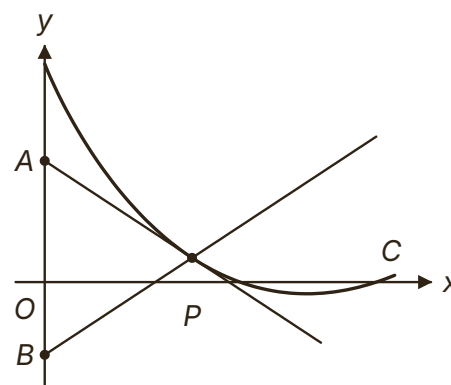
Answer

Total for D2: 5 marks

D3 The curve C has equation $y = x\sqrt{x} - 4x + 9$, $x > 0$. The point $P(4, 1)$ lies on C . The tangent to C at P meets the y -axis at A , and the normal to C at P meets the y -axis at B , as shown.

(a) Find $\frac{dy}{dx}$. [2 marks]

Answer



Not drawn accurately

(b) Show that the tangent to C at P has equation $y = 5 - x$.

[3 marks]

(c) Find an equation of the normal to C at P .

[2 marks]

Answer $y = \dots\dots$

(d) Find the area of triangle APB .

[3 marks]

Answer

Total for D3: 10 marks

D4 The curve C has equation $y = ax^3 + bx^2 - 4$, where a and b are constants. The point $(2, 8)$ lies on C , and the normal to C at $(2, 8)$ is parallel to the line $x + 4y = 3$. Find the value of a and the value of b .

[5 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The point P on the curve $y = x^3$ has x -coordinate p , where $p \neq 0$. [6 marks]
The tangent to the curve at P meets the curve again at Q . Find the x -coordinate of Q in terms of p , and show that the gradient of the curve at Q is always the same multiple of the gradient at P . State this multiple.

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Write down the tangent at (p, p^3) , then solve it with $y = x^3$. You already know one root, and it is a repeated root.
- Hint 2** The tangent is $y = 3p^2x - 2p^3$, so $x^3 - 3p^2x + 2p^3 = 0$, and $(x - p)^2$ is a factor.
- Hint 3** $x^3 - 3p^2x + 2p^3 = (x - p)^2(x + 2p)$, so Q has $x = -2p$. The gradient at Q is $3(-2p)^2 = 12p^2$, and at P it is $3p^2$. So it is always 4 times as big.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can find the equation of a tangent and of a normal to a curve. ○ ○ ○
- I can find the points where a tangent has a given gradient. ○ ○ ○
- I can find where a tangent or normal meets the curve again. ○ ○ ○
- I can find unknown constants from a tangent or normal. ○ ○ ○