

Graphs of sine, cosine and tangent

70 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-graphs-of-sine-cosine-and-tangent/

Date _____

A Do Now

Quick questions to start.

- | | |
|--|--|
| <p>A1 [Last lesson] [2 marks]</p> <p>A triangle has sides 6 cm and 9 cm with an angle of 50° between them. Find its area, to 3 s.f.</p> | <p>A2 [Needed today] [2 marks]</p> <p>Write down the exact values of $\cos 30^\circ$ and $\tan 45^\circ$.</p> |
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Answer and

Answer

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|---|---|
| <p>A3 [From chapter 4] [2 marks]</p> <p>The curve $y = f(x)$ has a maximum point at $(2, 5)$. Write down the maximum point on $y = f(x - 3)$ and on $y = 2f(x)$.</p> | <p>A4 [From chapter 9] [2 marks]</p> <p>In triangle ABC, $BC = 7$ cm, angle $A = 40^\circ$ and angle $B = 75^\circ$. Find AC, to 3 s.f.</p> |
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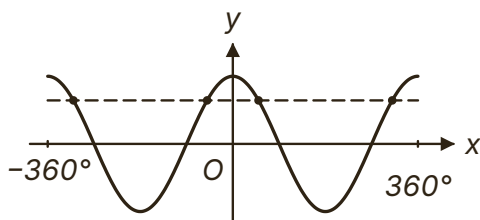
Answer

Answer and

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$\cos 50^\circ = 0.643$ to 3 s.f. Use the graph of $y = \cos x$ to find all the values of x in $-360^\circ \leq x \leq 360^\circ$ for which $\cos x = 0.643$.



Hint $y = \cos x$ is symmetrical about the y -axis and repeats every 360° .

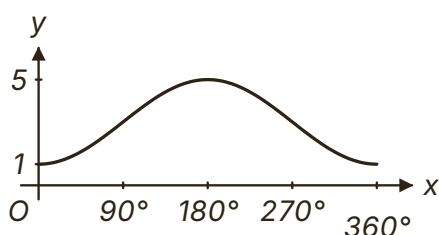
B1 $\tan 35^\circ = 0.700$ to 3 s.f. [3 marks]

Use the graph of $y = \tan x$ to find all the values of x in $-180^\circ \leq x \leq 360^\circ$ for which $\tan x = -0.700$.

Answer $x = \dots\dots$

EXAMPLE 2

For $y = 3 - 2 \cos x$, $0^\circ \leq x \leq 360^\circ$, find the maximum and minimum values of y and where they occur. State where the curve meets the y -axis.



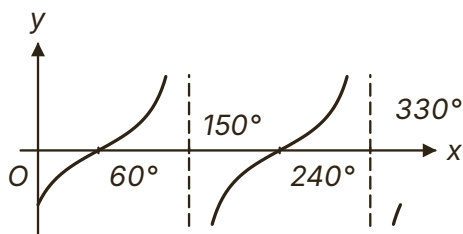
Hint Because of the minus sign, the largest y comes from the smallest $\cos x$.

B2 For $y = 2 - 5 \sin x$, [3 marks]

$-180^\circ \leq x \leq 180^\circ$, find the maximum and minimum values of y and where they occur. State where the curve meets the y -axis.

EXAMPLE 3

Sketch $y = \tan(x - 60^\circ)$,
 $0^\circ \leq x \leq 360^\circ$. Give the asymptotes
 and the points where the curve meets
 the axes.



Hint $y = \tan(x - 60^\circ)$ is $y = \tan x$ moved
 60° to the right. Move the asymptotes too.

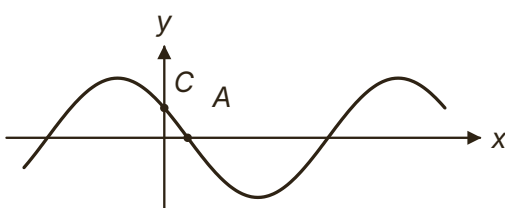
B3 Sketch

[4 marks]

$y = \tan(x + 30^\circ)$,
 $0^\circ \leq x \leq 360^\circ$. Give the
 asymptotes and the
 exact points where the
 curve meets the axes.

EXAMPLE 4

Part of $y = \sin(x + k)$, $0^\circ < k < 180^\circ$,
 is shown. It meets the x -axis going
 down at $A(30^\circ, 0)$ and the y -axis at C .
 Find k and C exactly.



Not drawn accurately

Hint $\sin \theta$ crosses zero going down when
 $\theta = 180^\circ$. So $30^\circ + k = 180^\circ$.

B5

[4 marks]

$y = \cos(x + k)$,
 $0^\circ < k < 180^\circ$, has a
 minimum point at
 $P(135^\circ, -1)$. Find k , the
 exact point where the
 curve meets the y -axis,
 and the maximum point
 for $0^\circ \leq x \leq 360^\circ$.

- C1** Describe fully the single transformation that maps (a) $y = \sin x$ onto $y = \sin(x - 20^\circ)$; (b) $y = \cos x$ onto $y = \cos \frac{1}{3}x$; (c) $y = \tan x$ onto $y = \tan x - 2$; (d) $y = \sin x$ onto $y = \sin(-x)$. [4 marks]

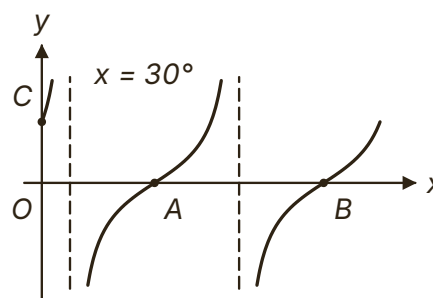
- C2** Sketch $y = -2 \cos x$ for $-180^\circ \leq x \leq 180^\circ$. Give the coordinates of the points where the curve meets the axes, and of its maximum and minimum points. [4 marks]

- C3** $f(x) = 3 \sin 2x + 1$. Write down the period of f and the range of f . Find how many times the graph of $y = f(x)$ crosses the x -axis for $0^\circ \leq x \leq 360^\circ$. [3 marks]

Answer period = ... range times

D1 The diagram shows the curve $y = \tan(x + k)$, $0^\circ \leq x \leq 360^\circ$, where $0^\circ < k < 90^\circ$. The line $x = 30^\circ$ is an asymptote. The curve meets the x -axis at A and B , and the y -axis at C .

- (a) Find the value of k , and write down the equation of the other asymptote for $0^\circ \leq x \leq 360^\circ$. [3 marks]



Not drawn accurately

Answer $k = \dots\dots\dots$ $x = \dots\dots\dots$

- (b) Find the coordinates of A , B and C , giving C exactly. [4 marks]

- (c) Hence solve $\tan(x + k) = 1$ for $0^\circ \leq x \leq 360^\circ$. [3 marks]

Answer $x = \dots\dots\dots$

Total for D1: 10 marks

D2

Ali writes: "The graph of $y = \sin 2x$ is the graph of $y = \sin x$ stretched parallel to the x -axis with scale factor 2. So the period of $y = \sin 2x$ is 720° ."

Ali is wrong.

- (a) Explain Ali's mistake and give the correct period. Write down the period of $y = \sin \frac{1}{2}x$. [3 marks]

- (b) One solution of $\sin 2x = 0.3$ is $x = 8.73^\circ$. Using the symmetry and the period of the graph of $y = \sin 2x$, find the other three solutions for $0^\circ \leq x \leq 360^\circ$. [3 marks]

Answer $x = \dots\dots$

Total for D2: 6 marks

- D4** A big wheel turns at a steady speed. The height, h metres, of a seat above the ground t minutes after it passes the lowest point is modelled by $h = 20 - 18 \cos(15t)^\circ$, $0 \leq t \leq 48$.

- (a) Write down the greatest height of the seat and the first time it reaches it. Find the time for one complete turn of the wheel. [3 marks]

Answer $\dots\dots$ m at $t = \dots\dots$ one turn $\dots\dots$ minutes

- (b) Find the times during the first turn when the seat is at least 29 m [4 marks]
above the ground.

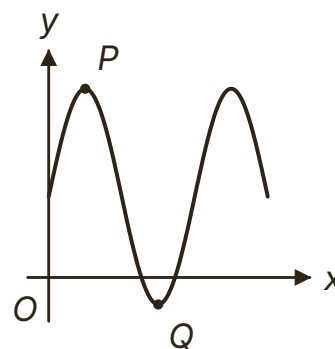
Answer $\dots \leq t \leq \dots$

- (c) Hence find the total time for $0 \leq t \leq 48$ that the seat is at least 29 [2 marks]
m above the ground. Give one reason why the model may not be
realistic.

Total for D4: 9 marks

- D5** The diagram shows part of the curve $y = a \sin bx + c$, where a , b and c are
positive constants. $P(15^\circ, 7)$ is a maximum point and $Q(45^\circ, -1)$ is the next
minimum point.

- (a) Find the values of a , b and c . [4 marks]



Not drawn accurately

Answer $a = \dots$ $b = \dots$ $c = \dots$

- (b) Hence find the number of solutions of $a \sin bx + c = 5$ for $0^\circ \leq x \leq 360^\circ$.

[2 marks]

Answer

Total for D5: 6 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Find all the solutions of $\sin 5x = \sin x$ for $0^\circ \leq x \leq 180^\circ$. Do not use a calculator. [6 marks]

Answer $x = \dots\dots$

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Look at the graph of $y = \sin \theta$. Two angles have the same sine when they are equal, or when they add up to 180° , give or take whole turns of 360° .
- Hint 2** So $5x = x + 360^\circ n$ or $5x = 180^\circ - x + 360^\circ n$, for whole numbers n .
- Hint 3** The first gives $x = 90^\circ n$: $0^\circ, 90^\circ, 180^\circ$. The second gives $6x = 180^\circ + 360^\circ n$, so $x = 30^\circ + 60^\circ n$: $30^\circ, 90^\circ, 150^\circ$. Together: $0^\circ, 30^\circ, 90^\circ, 150^\circ, 180^\circ$

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can sketch the graphs of sin, cos and tan and mark their key points. ☐ ☐ ☐

I can use symmetry and period to find more values from one. ☐ ☐ ☐

I can sketch a stretched or translated trig graph. ☐ ☐ ☐

I can find the constants in a trig graph from its key points. ☐ ☐ ☐