Writing a cos x ± b sin x as R cos(x ± α)

119 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-harmonic-form-r-cos-x-alpha/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Expand $R\cos(x-\alpha)$.

Answer .....

A2 From chapter 6 [2 marks]

The angle α is acute and $\tan\alpha = \frac{3}{4}$. Find α to 1 decimal place and the exact value of $\cos\alpha$.

Answer .....

A3 Common slip [2 marks]

Find the greatest value of $5 - 2\cos x$, and the value of x in $0° \leq x < 360°$ where it happens.

Answer .....

A4 Today's idea [3 marks]

$R\cos\alpha = 8$ and $R\sin\alpha = 15$, where $R > 0$ and α is acute. Find R, and α to 1 decimal place.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Write $\sqrt{3}\cos x + \sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

Hint Expand R cos(x – α) and match the coefficients of cos x and of sin x.

B1 Write $5\cos x + 12\sin x$ in the form $R\cos(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give α to 2 decimal places. [5 marks]

Answer .....

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EXAMPLE 2

Write $2\sin x - 3\cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$.

Hint R sin(x − α) = R sin x cos α − R cos x sin α. The minus sign is already in the form, so R cos α and R sin α are both positive.

B2 Write $4\sin x - \cos x$ in the form $R\sin(x - \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give R exactly and α to 2 decimal places. [5 marks]

Answer .....

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EXAMPLE 3

Find the greatest and least values of $3\cos x + 4\sin x$, and the values of x in $0° \leq x < 360°$ where they happen.

Hint Write it as 5 cos(x – 53.13°). A cosine is greatest when its angle is 0° and least when its angle is 180°.

B3 [5 marks]

$f(x) = \frac{20}{15 - 5\cos x - 12\sin x}$. Use your answer to B1 to find the greatest value of $f(x)$, and the smallest positive value of x where it happens.

Answer .....

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EXAMPLE 4

Solve $\sqrt{3}\cos x + \sin x = 1$ for $0° \leq x < 360°$.

Hint Use 2 cos(x – 30°) from Example 1. x – 30° runs from −30° to 330°.

B4 Solve $3\sin x - 4\cos x = 2$ for $0° \leq x < 360°$. Give your answers to 1 decimal place. [5 marks]

Answer .....

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EXAMPLE 5

Solve $\sin 2\theta + \cos 2\theta = 1$ for $0 \leq \theta < \pi$.

Hint Write it as √2 sin(2θ + π/4). Then 2θ + π/4 runs from π/4 to 9π/4.

B5 Solve $2\cos 2\theta - 3\sin 2\theta = 1$ for $0 \leq \theta < \pi$. Give your answers to 3 significant figures. [5 marks]

Answer .....

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C1 Leo writes $6\cos x + 8\sin x = 10\cos(x + 53.1^\circ)$. Explain his mistake and write the expression correctly. [4 marks]

C2 $g(x) = 7 + 2\sqrt{3}\cos x - 2\sin x$. Find the range of g, and the smallest positive value of x for which $g(x)$ is greatest. [5 marks]

Answer .....

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C3 State the number of solutions of $3\cos x + 4\sin x = k$ for $0° \leq x < 360°$ when (i) $k = 6$ (ii) $k = 5$ (iii) $k = 2$. Give a reason. [4 marks]

Answer .....

C4 Solve $8\cos x + 15\sin x = 10$ for $-180° < x < 180°$. Give your answers to 1 decimal place. [5 marks]

Answer .....

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C5 Write $\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta - \alpha)$, where $R > 0$ and $0 < \alpha < \dfrac{\pi}{2}$, with α exact. Hence find the least value of the expression and the value of θ in $0 \leq \theta < 2\pi$ where it happens. [4 marks]

Answer .....

D Exam-style questions

Edexcel exam style

D1

(a) Express $7\cos x - 24\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give α to 2 decimal places. [3 marks]

Answer .....

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(b) Hence find the greatest value of $7\cos x - 24\sin x + 3$, and the smallest positive value of x at which it happens. [3 marks]

Answer .....

(c) Solve $7\cos x - 24\sin x = 15$ for $0° \leq x < 360°$, giving your answers to 1 decimal place. [4 marks]

Answer .....

Total for D1: 10 marks

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D2

> Priya writes: "The greatest value of $\sin x$ is 1 and the greatest value of $\cos x$ is 1, so the greatest value of $3\sin x + 4\cos x$ is $3 + 4 = 7$."

Priya is asked for the greatest value of $3\sin x + 4\cos x$.

(a) Explain why Priya is wrong. [2 marks]

(b) Express $3\sin x + 4\cos x$ in the form $R\sin(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give α to 2 decimal places. [3 marks]

Answer .....

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(c) $h(x) = (3\sin x + 4\cos x)^2 + 6(3\sin x + 4\cos x)$. Find the greatest and least values of $h(x)$. [4 marks]

Answer .....

Total for D2: 9 marks

D3 Solve $\sin x + \sqrt{3}\cos x = \sqrt{2}$ for $0 \leq x < 2\pi$. Give exact answers. [7 marks]

Answer .....

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D4

(a) Show that $4\sin^2 x + 6\sin x \cos x$ can be written as $2 - 2\cos 2x + 3\sin 2x$. [3 marks]

(b) Express $3\sin 2x - 2\cos 2x$ in the form $R\sin(2x - \alpha)$, where $R > 0$ and $0^\circ < \alpha < 90^\circ$. Give R exactly and α to 2 decimal places. [3 marks]

Answer .....

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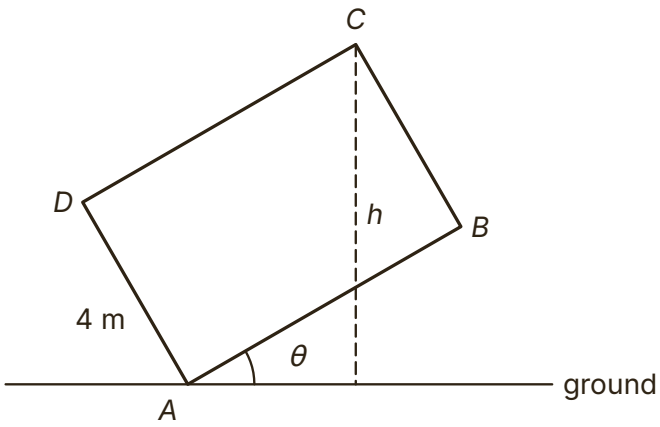
(c) Hence state the greatest value of $4\sin^2 x + 6\sin x \cos x$, and solve $4\sin^2 x + 6\sin x \cos x = 3$ for $0^\circ \leq x < 180^\circ$, giving your answers to 1 decimal place. [5 marks]

Answer .....

Total for D4: 11 marks

D5 A rectangular panel $ABCD$ has $AB = 6$ m and $AD = 4$ m. Corner A rests on horizontal ground and AB makes an angle θ with the ground, where $0^\circ < \theta < 90^\circ$. The panel is in a vertical plane. h m is the height of C above the ground. The figure is not drawn to scale.

(a) Show that $h = 6\sin\theta + 4\cos\theta$. [2 marks]



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(b) Express h in the form $R\sin(\theta + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. Give R exactly and α to 2 decimal places. [3 marks]

Answer .....

(c) Find the greatest possible height of C, and the value of θ that gives it. [2 marks]

Answer .....

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(d) Find the values of θ for which $h = 7$, giving your answers to 1 decimal place. [3 marks]

Answer .....

Total for D5: 10 marks

D6 $f(x) = \dfrac{12}{3 + \cos x - \sqrt{3}\sin x}$, $0° \leq x < 360°$.

(a) Express $\cos x - \sqrt{3}\sin x$ in the form $R\cos(x + \alpha)$, where $R > 0$ and $0° < \alpha < 90°$. [2 marks]

Answer .....

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(b) Find the range of f. [3 marks]

Answer .....

(c) Solve $f(x) = 4$. [3 marks]

Answer .....

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Find the greatest and least values of $5\sin^2 x + 6\sin x\cos x - 3\cos^2 x$, and the values of x in $0 \leq x < \pi$ where they happen. Give the values of x to 3 significant figures. [8 marks]

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Make every term a function of $2x$: use the double-angle formulae for $\sin^2 x$, $\cos^2 x$ and $\sin x \cos x$.

Hint 2 You should get $1 - 4\cos 2x + 3\sin 2x$. Write $3\sin 2x - 4\cos 2x$ as one sine.

Hint 3 $5\sin(2x - 0.927)$ is greatest when its angle is $\frac{\pi}{2}$ and least when it is $\frac{3\pi}{2}$, since $2x - 0.927$ runs from -0.927 to $2\pi - 0.927$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can write a cos x + b sin x as R cos(x − α) or R sin(x + α), with the right sign of α. ○ ○ ○
- I can find R from R² = a² + b² and α from tan α, choosing the quadrant from R cos α and R sin α. ○ ○ ○
- I can give the greatest and least values of an expression and the x where they happen. ○ ○ ○
- I can solve a cos x + b sin x = c by moving the interval for x ± α first. ○ ○ ○

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