Implicit differentiation

113 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-implicit-differentiation/

Date

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A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Differentiate x^2e^{3x} with respect to x.

Answer .....

A2 Needed today [2 marks]

A line has gradient $\frac{2}{5}$.

Write down the gradient of a line perpendicular to it, and find the equation of that perpendicular line through $(1, 3)$.

Answer .....

A3 Common slip [2 marks]

y is a function of x. Find $\frac{d}{dx}\left(y^3\right)$ in terms of y and $\frac{dy}{dx}$.

Answer .....

A4 Common slip [2 marks]

y is a function of x. Find $\frac{d}{dx}(4xy)$.

Answer .....

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A5 From chapter 9 [2 marks]

Write 3^x in the form e^{kx}, and hence differentiate 3^x.

Answer .....

B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A curve has equation $x^2 + 3xy + y^2 = 11$. Find $\frac{dy}{dx}$ in terms of x and y.

Hint Differentiate every term with respect to x. $3xy$ needs the product rule, and y^2 gives $2y\frac{dy}{dx}$.

B1 A curve has equation $x^3 + 2xy - y^2 = 7$. Find $\frac{dy}{dx}$ in terms of x and y. [4 marks]

Answer .....

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EXAMPLE 2

The curve $x^2 + xy - y^2 = 11$ passes through $(3, 2)$. Find an equation of the normal to the curve at $(3, 2)$. Give it in the form $ax + by + c = 0$.

Hint Differentiate, then put $x = 3$ and $y = 2$ in before you rearrange: it is quicker.

B2 The curve $2x^2 - xy + y^2 = 16$ passes through $(2, 4)$. Find an equation of the tangent to the curve at $(2, 4)$. Give it in the form $ax + by + c = 0$, where a, b and c are integers. [5 marks]

Answer .....

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EXAMPLE 3

A curve has equation $xe^y + y = 1$. Find the gradient of the curve at the point $(1, 0)$.

Hint xe^y is a product. The derivative of e^y with respect to x is $e^y \dfrac{dy}{dx}$.

B3 A curve has equation $e^{xy} + 3y = 4$. Find the gradient of the curve at the point $(0, 1)$. [4 marks]

Answer .....

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EXAMPLE 4

Find the coordinates of the stationary points on the curve $x^2 - xy + y^2 = 12$.

Hint Stationary means $\frac{dy}{dx} = 0$, so the top of the fraction is zero. That gives a link between x and y: put it back into the curve.

B4 Find the coordinates of the stationary points on the curve $x^2 + 2xy + 3y^2 = 18$. [5 marks]

Answer .....

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EXAMPLE 5

Find the coordinates of the points on the curve $x^2 + xy + y^2 = 27$ where the tangent is vertical.

Hint A vertical tangent means the bottom of $\frac{dy}{dx}$ is zero.

B5 Find the coordinates of the points on the curve $x^2 - 4xy + 5y^2 = 4$ where the tangent is vertical. Give exact answers. [5 marks]

Answer .....

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C1 A curve has equation $x^2 + 3xy + y^3 = 5$. Ali differentiates and writes $2x + 3\dfrac{dy}{dx} + 3y^2\dfrac{dy}{dx} = 0$. Explain his mistake, and find the correct gradient at $(1, 1)$. [5 marks]

Answer .....

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C2 A curve has equation $x^2 + y^2 + xy = 7$. Beth differentiates the 7 and gets 7, so her equation is $2x + 2y\dfrac{dy}{dx} + y + x\dfrac{dy}{dx} = 7$. Explain her mistake, and find the correct gradient at $(1, 2)$. [4 marks]

Answer .....

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C3 For the curve $x^2 + 4xy + 5y^2 = 4$, Chen finds $\dfrac{dy}{dx} = -\dfrac{x + 2y}{2x + 5y}$. [5 marks]

He says: "The stationary points are where $x = -2y$, so there are infinitely many." Explain his mistake, and find the stationary points.

Answer .....

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C4 A curve has equation $e^{2y} + xy = 1 + x$. Dev differentiates e^{2y} and gets $2e^{2y}$. Explain his mistake, and find an equation of the tangent to the curve at the origin. [5 marks]

Answer .....

D Exam-style questions

Edexcel exam style

D1 A curve C has equation $3x^2 - 2xy + y^2 = 9$. The point $P(2, 3)$ lies on C.

(a) Find $\dfrac{dy}{dx}$ in terms of x and y. [3 marks]

Answer .....

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(b) Find an equation of the normal to C at P. Give it in the form $ax + by + c = 0$, where a, b and c are integers. [4 marks]

Answer .....

(c) The normal at P meets C again at Q. Find the coordinates of Q. [4 marks]

Answer .....

Total for D1: 11 marks

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D2

A curve has equation $x^2 + 4y^2 - 2xy = 12$. Ravi differentiates and writes: "$2x + 8y\frac{dy}{dx} - 2y = 0$, so $\frac{dy}{dx} = \frac{y - x}{4y}$."

Ravi is wrong.

(a) Explain Ravi's mistake, and find the correct $\frac{dy}{dx}$. [3 marks]

Answer .....

(b) Find the coordinates of the stationary points on the curve. [3 marks]

Answer .....

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(c) Find the coordinates of the points where the tangent to the curve is vertical. [4 marks]

Answer .....

Total for D2: 10 marks

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D3 A curve has equation $2^x + 2^y = 6$. Find the gradient of the curve at the point $(2, 1)$. [6 marks]

Answer .....

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D4 A curve has equation $x^2 \tan y = 9$, where $x > 0$ and $0 < y < \frac{\pi}{2}$.

(a) Show that $\frac{dy}{dx} = -\frac{18x}{x^4 + 81}$. [4 marks]

(b) Find an equation of the tangent to the curve at the point where $x = 3$. Give it in the form $y = mx + c$, where m and c are exact. [4 marks]

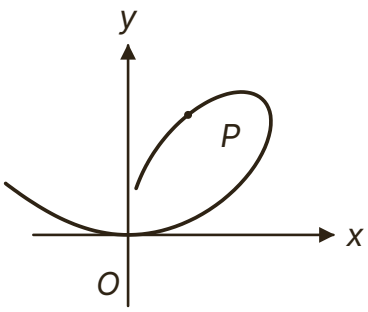
Answer .....

Total for D4: 8 marks

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D5 The diagram shows the curve with equation $x^3 + y^3 = 9xy$. The point $P(2, 4)$ lies on the curve.

(a) Find the gradient of the curve at P. [4 marks]



Answer .....

(b) The normal to the curve at P meets the x-axis at A and the y-axis at B. Find the area of triangle OAB. [5 marks]

Answer .....

Total for D5: 9 marks

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D6 This question is about the curve $y = 4^x - 2^{x+1}$.

(a) By taking logarithms of $y = 4^x$, show that $\frac{d}{dx}(4^x) = 4^x \ln 4$. [2 marks]

(b) Find the coordinates of the stationary point on the curve $y = 4^x - 2^{x+1}$. [3 marks]

Answer .....

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(c) Find the exact gradient of the curve at the point where $x = 1$. [3 marks]
Give it in the form $k \ln 2$.

Answer .....

Total for D6: 8 marks

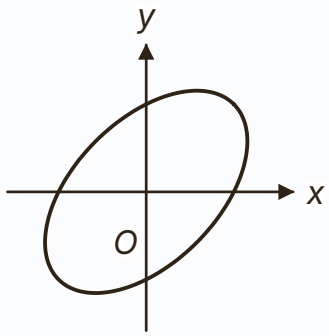
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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The diagram shows the curve with equation $x^2 - xy + y^2 = 3$. [9 marks]
Find the greatest value of y on the curve. Then find the equations of the two tangents to the curve that are parallel to the line $y = x$.



Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Differentiate implicitly: $\dfrac{dy}{dx} = \dfrac{2x - y}{x - 2y}$.

Hint 2 At the greatest y the tangent is horizontal, so $y = 2x$. Put that back into the curve.

Hint 3 Parallel to $y = x$ means $\dfrac{dy}{dx} = 1$. That gives $x = -y$; put it back into the curve.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can differentiate a term in y with respect to x by multiplying by dy/dx.	○ ○ ○
I can use the product rule on a term such as xy when differentiating implicitly.	○ ○ ○
I can find gradients, tangents, normals and stationary points of a curve given implicitly.	○ ○ ○
I can differentiate an exponential with any base, such as 3 to the power x.	○ ○ ○

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