

Increasing and decreasing functions, and second derivatives

73 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-increasing-and-decreasing-functions/

Date _____

A Do Now

Quick questions to start.

- | | |
|---|--|
| <p>A1 Needed today [2 marks]</p> <p>Solve the inequality
$x^2 - 2x - 15 < 0$.</p> | <p>A2 Needed today [2 marks]</p> <p>Write $x^2 + 6x + 14$ in the
form $(x + a)^2 + b$.</p> |
|---|--|

Answer

Answer

- | | |
|--|--|
| <p>A3 Last lesson [2 marks]</p> <p>Find an equation of the
tangent to the curve
$y = x^2 - 5x + 1$ at the
point where $x = 4$.</p> | <p>A4 From chapter 2 [2 marks]</p> <p>The equation
$x^2 + kx + 9 = 0$ has no
real roots. Find the set of
possible values of k.</p> |
|--|--|

Answer $y = \dots\dots$

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find the interval on which the function $f(x) = 2x^3 - 9x^2 - 24x + 5$ is decreasing.

Hint f is decreasing where $f'(x) < 0$.
Factorise $f'(x)$ and solve the quadratic inequality.

B1 Find the set of values of x for which the function $f(x) = x^3 + 6x^2 - 36x + 2$ is increasing. [4 marks]

Answer

EXAMPLE 2

Show that the function $f(x) = x^3 - 3x^2 + 5x - 2$ is increasing for all real values of x .

Hint Differentiate, then complete the square on $f'(x)$ to show it is always positive.

B2 Show that the function $f(x) = 1 - 4x + 3x^2 - x^3$ is decreasing for all real values of x . [4 marks]

EXAMPLE 3

Given that $y = \frac{2x^4 - 3}{x}$, find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

Hint Split the fraction into powers of x first. Then differentiate twice.

B3 Given that [4 marks]

$$f(x) = 8\sqrt{x} + \frac{2}{\sqrt{x}},$$

$x > 0$, find $f''(x)$.

Hence find the value of $f''(4)$.

Answer $f''(4) = \dots\dots$

C Practice

A-Level

C1 Find the set of values of x for which $f(x) = x - 6\sqrt{x}$, $x > 0$, is decreasing. [4 marks]

Answer

C2 Given that $y = (x^2 - 3)(2x + 1)$, find the value of x for which $\frac{d^2y}{dx^2} = 0$. [3 marks]

Answer $x = \dots\dots$

- C3** $f(x) = ax^3 + bx^2 - 5x$, where a and b are constants. Given that $f'(2) = 7$ and $f''(2) = 18$, find the value of a and the value of b . [4 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

- C4** The function $f(x) = x^3 - px$, where p is a constant, is decreasing on the interval $[-2, 2]$. Find the set of possible values of p . [3 marks]

Answer $p \dots\dots$

D Exam-style questions

Edexcel exam style

- D1** Leah is asked to show that $f(x) = x^3 - 6x^2 + 15x - 2$ is increasing for all real x . She writes: " $f'(x) = 3x^2 - 12x + 15$. $f'(0) = 15 > 0$ and $f'(5) = 30 > 0$, so f is increasing for all x ."

Leah's argument is not correct.

- (a) Explain why Leah's working does not show that f is increasing for all x . [2 marks]

(b) Show that f is increasing for all real values of x . [3 marks]

(c) Find $f''(x)$. Hence find the least value of the gradient of the curve $y = f(x)$. [3 marks]

Answer least gradient = ...

Total for D1: 8 marks

D2 The function f is defined by $f(x) = x^4 - 18x^2 + 5$.

(a) Find $f'(x)$ and $f''(x)$. [3 marks]

Answer

(b) Find the set of values of x for which f is decreasing. [4 marks]

Answer

- (c) Hence find the set of values of x for which f is decreasing and $f''(x) > 0$. [3 marks]

Answer

Total for D2: 10 marks

D3 $f(x) = x^3 + 3kx^2 + 3(k + 6)x - 1$, where k is a constant.

- (a) Show that $f'(x) = 3(x^2 + 2kx + k + 6)$. [2 marks]

- (b) Given that $f'(x) > 0$ for all real values of x , so that f is increasing for all x , find the set of possible values of k . [4 marks]

Answer

- (c) In the case $k = 3$, show that $f'(x) \geq 0$ for all x , and find the value of x for which $f'(x) = 0$. [2 marks]

Answer $x = \dots\dots$

Total for D3: 8 marks

- D4** A particle moves along a straight line. Its displacement from a fixed point O at time t seconds is s metres, where $s = t^3 - 12t^2 + 45t$, $t \geq 0$. Its velocity is $v = \frac{ds}{dt}$ and its acceleration is $a = \frac{d^2s}{dt^2}$.

- (a) Find expressions for v and a in terms of t . [2 marks]

Answer

- (b) Find the times at which the particle is at rest. [2 marks]

Answer $t = \dots\dots$ and $t = \dots\dots$

- (c) Find the interval of time during which the velocity of the particle is decreasing. Hence find the least value of the velocity. [3 marks]

Answer interval ; least velocity m s^{-1}

Total for D4: 7 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** $f(x) = kx^3 - 3x^2 + kx + 2$, where k is a non-zero constant. Find the set of values of k for which $f'(x) < 0$ for every real value of x , so that f is decreasing for all x . [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** $f'(x)$ is a quadratic in x . A quadratic is negative for every x only if its graph is an upside-down U that never reaches the x -axis.
- Hint 2** $f'(x) = 3kx^2 - 6x + k$. You need two things: $3k < 0$, and $3kx^2 - 6x + k = 0$ has no real roots.
- Hint 3** $k < 0$, and the discriminant $36 - 12k^2 < 0$, so $k^2 > 3$. With $k < 0$ this gives $k < -\sqrt{3}$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find where a function is increasing or decreasing. ☐ ☐ ☐

I can show a function is increasing or decreasing for all x . ☐ ☐ ☐

I can find and use a second derivative. ☐ ☐ ☐

I can use derivatives for velocity and acceleration. ☐ ☐ ☐