



Index laws, including negative and fractional indices

74 marks

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] **A2** GCSE Higher [2 marks]
Evaluate $27^{\frac{2}{3}}$. Simplify $(3a^2b)^3$.

Answer

Answer

- A3** GCSE Higher [2 marks] **A4** GCSE Higher [2 marks]
Expand and simplify Solve $2^x = \frac{1}{8}$.
 $(2x - 3)(x + 4)$.

Answer

Answer $x = \dots\dots$

B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Simplify $\frac{(4x^5y^2)^2}{2x^3y}$.

Hint Square the 4, the x^5 and the y^2 separately. Then divide the numbers, and subtract the powers of x and of y .

- B1** [3 marks]
Simplify
 $(3a^4b^3)^2 \times 2a^{-1}b$.

Answer

EXAMPLE 2

Evaluate (a) $216^{-\frac{2}{3}}$ (b) $\left(\frac{8}{125}\right)^{-\frac{2}{3}}$

Hint A negative power means one over. The bottom of the fraction in the power is the root; the top is the power. Take the root first: the numbers stay small.

B2 Evaluate (a) $128^{-\frac{3}{7}}$ [4 marks]

(b) $\left(\frac{100}{9}\right)^{-\frac{3}{2}}$

Answer (a) (b)

EXAMPLE 3

Write $\frac{3x^2 - 4\sqrt{x}}{2x}$ in the form $ax^p + bx^q$, where a , b , p and q are constants.

Hint Write $\sqrt{x}$ as $x^{\frac{1}{2}}$, then divide each term on top by $2x$ separately.

B3 Write $\frac{5x^3 + 2\sqrt{x}}{x\sqrt{x}}$ in the [3 marks]

form $ax^p + bx^q$.

Answer

EXAMPLE 4

Write $8^{x+1} \times 4^{2x}$ as a single power of 2.

Hint $8 = 2^3$ and $4 = 2^2$. Then $(2^3)^{x+1} = 2^{3x+3}$.

B4 Write $\frac{9^x}{27^{x-2}}$ as a single [3 marks]

power of 3.

Answer

EXAMPLE 5

Solve $4^{x+2} = 8^{2x-1}$.

Hint Write both sides as powers of 2. If $2^m = 2^n$ then $m = n$.

B5 Solve $9^{x+1} = \frac{27^x}{3}$. [3 marks]

Answer $x = \dots\dots$

c Practice

A-Level

C1 Simplify $(2x^{\frac{1}{2}})^3 \times 3x^{-\frac{1}{2}}$. [2 marks]

Answer

C2 Write $\frac{(2 + \sqrt{x})^2}{x}$ in the form $ax^{-1} + bx^{-\frac{1}{2}} + c$, where a , b and c are integers. [3 marks]

Answer

C3 Given that $y = 3^x$, write $9^{x+1} - 3^{x-1}$ in terms of y .

[3 marks]

Answer

C4 Solve $\left(\frac{1}{16}\right)^x = 8^{x+2}$.

[3 marks]

Answer $x = \dots\dots$

D Exam-style questions

Edexcel exam style

D1 Given that $y = \frac{1}{8}x^3$, express each of the following in the form kx^n , where k and n are constants.

(a) $y^{\frac{1}{3}}$

[2 marks]

Answer

(b) $\frac{1}{2}y^{-\frac{2}{3}}$

[3 marks]

Answer

Total for D1: 5 marks

D2 $f(x) = \frac{(2\sqrt{x} - 3)^2}{x\sqrt{x}}, \quad x > 0$

(a) Show that $f(x)$ can be written as $4x^{-\frac{1}{2}} - 12x^{-1} + 9x^{-\frac{3}{2}}$. [4 marks]

(b) Hence find the exact value of $f(9)$. [2 marks]

Answer

(c) Solve $f(x) = 0$. [2 marks]

Answer $x = \dots\dots$

Total for D2: 8 marks

D3

Ravi writes: $\left(\frac{4}{x^2}\right)^{-\frac{3}{2}} = 4 \times \left(-\frac{3}{2}\right) \times x^3 = -6x^3$

Ravi is wrong.

(a) Describe the two mistakes Ravi has made. [2 marks]

(b) Write $\left(\frac{4}{x^2}\right)^{-\frac{3}{2}}$ in the form kx^n . [3 marks]

(c) Hence solve $\left(\frac{4}{x^2}\right)^{-\frac{3}{2}} = 27$, where $x > 0$. [3 marks]

Answer

Answer $x = \dots\dots$

Total for D3: 8 marks

D4 Given that $y = 2^x$

(a) write 4^x in terms of y . [1 mark]

Answer

- (b) Show that the equation $2^{2x+1} - 17 \times 2^x + 8 = 0$ can be written as $2y^2 - 17y + 8 = 0$. [2 marks]

- (c) Hence solve $2^{2x+1} - 17 \times 2^x + 8 = 0$. [4 marks]

Answer $x = \dots\dots$ or $x = \dots\dots$

Total for D4: 7 marks

-
- D5** Find the exact value of x for which $\frac{9^x}{3^{1-x}} = 27\sqrt{3}$.

- (a) Write $27\sqrt{3}$ as a power of 3. [2 marks]

Answer

- (b) Hence find x . [3 marks]

Answer $x = \dots\dots$

Total for D5: 5 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** Find every real number x for which $(x^2 - 5x + 5)^{x^2 - 9x + 20} = 1$, and [6 marks]
work out their sum.

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** A power equals 1 in three ways: the base is 1; the power is 0 (and the base is not 0); or the base is -1 and the power is even.
- Hint 2** Base 1: $x^2 - 5x + 4 = 0$. Power 0: $x^2 - 9x + 20 = 0$. Base -1 : $x^2 - 5x + 6 = 0$, then check the power is even.
- Hint 3** Base 1 gives $x = 1, 4$. Power 0 gives $x = 4, 5$ (the base is not 0 there). Base -1 gives $x = 2, 3$, where the power is 6 and 2, both even. So $x = 1, 2, 3, 4, 5$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use the index laws with negative and fractional powers. ○ ○ ○

I can write an expression as a sum of powers of x . ○ ○ ○

I can write numbers as powers of one base to solve equations. ○ ○ ○

I can spot a quadratic in disguise when $y = a^x$. ○ ○ ○