

Inequalities on graphs and regions

70 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-inequalities-on-graphs-and-regions/

Date _____

A Do Now

Quick questions to start.

- | | |
|---|---|
| <p>A1 Needed today [2 marks]</p> <p>Find the point where the lines $y = 3x - 4$ and $x + y = 8$ meet.</p> | <p>A2 Last lesson [2 marks]</p> <p>Find the set of values of x for which $x^2 - 2x - 24 > 0$.</p> |
|---|---|

Answer (....., ...)

Answer

- | | |
|---|---|
| <p>A3 From chapter 2 [2 marks]</p> <p>Work out the discriminant of $2x^2 - 5x + 4$. How many real roots does $2x^2 - 5x + 4 = 0$ have?</p> | <p>A4 From chapter 1 [2 marks]</p> <p>Write $\sqrt{98} + \sqrt{18}$ in the form $k\sqrt{2}$.</p> |
|---|---|

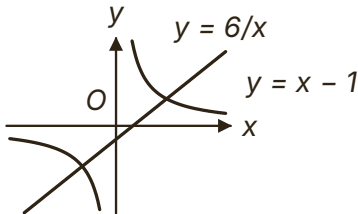
Answer

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Use the sketch to solve $\frac{6}{x} < x - 1$, $x \neq 0$, giving your answer in set notation.



Hint Find where the graphs meet. The curve has two branches, so $x = 0$ is a critical value too.

B1 By sketching $y = \frac{8}{x}$ and $y = x + 2$ on the same axes, find, in set notation, the set of values of x for which $\frac{8}{x} > x + 2$, $x \neq 0$. [4 marks]

Answer

EXAMPLE 2

Draw on a grid the region R that satisfies $y > -1$, $y \leq x + 2$ and $x + y < 6$. Find the coordinates of the vertices of R and the area of R . Which vertices belong to R ?

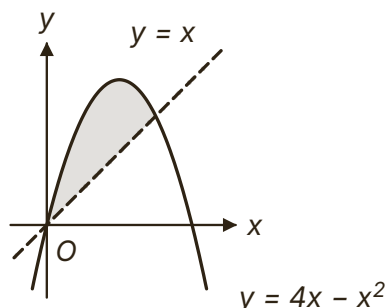
Hint A strict inequality ($<$ or $>$) has a dashed line; $\leq$ or $\geq$ has a solid line. A vertex belongs to R only if both lines through it are solid.

B2 The region R is the set of points with $y \geq -1$, $y < 2x + 1$ and $2x + y \leq 9$. Find the vertices of R and the area of R . [4 marks]

Answer

EXAMPLE 3

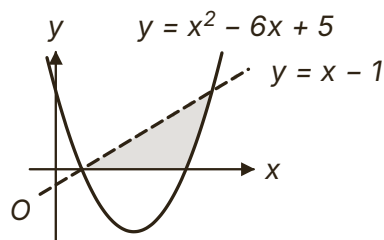
The sketch shows the curve $y = 4x - x^2$ and the line $y = x$. The line is dashed and the curve is solid. Write down the inequalities that define the shaded region R .



Hint Is R above or below each boundary? A solid boundary is part of R , so use $\leq$ or $\geq$ for it.

B3 [3 marks]

The sketch shows the curve $y = x^2 - 6x + 5$ and the line $y = x - 1$. The line is dashed; the curve and the x -axis are solid. Write down the inequalities that define the shaded region R .

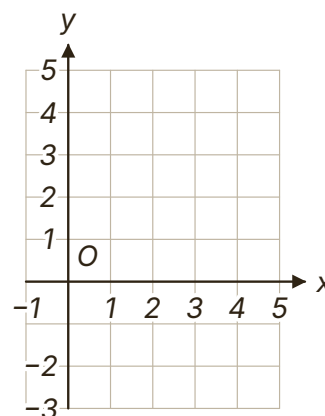


Answer

c Practice

A-Level

- C1** On the grid, shade the region R that satisfies $x \geq 0$, $y \geq x - 2$ and $x + y < 4$. Find the area of R , and state which vertices of R belong to R . [5 marks]

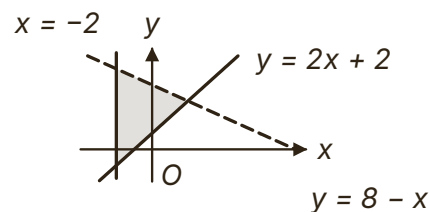


Answer

- C2** The region R is the set of points with $y \geq x^2 - 2x - 3$ and $y < x + 1$. [3 marks]
Which of the points $P(0, 0)$, $Q(4, 5)$ and $S(-1, 0)$ lie in R ? Give a reason for each.

Answer

- C3** The sketch shows the lines $x = -2$, $y = 2x + 2$ and $y = 8 - x$. The line $y = 8 - x$ is dashed; the other two are solid. Write down the inequalities that define the shaded region R , and find the area of R . [4 marks]



Answer

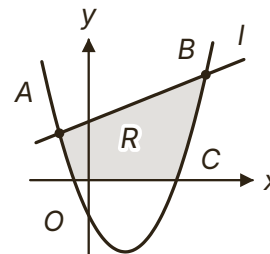
- C4** Find the set of values of k for which the curve $y = x^2 + kx + 9$ lies entirely above the line $y = x$. [4 marks]

Answer

- D1** The curve C has equation $y = 2x^2 - 5x - 3$ and the line l has equation $y = x + 5$. The line meets the curve at the points A and B , as shown. The region R , shown shaded, is bounded by C , l and the x -axis, and includes its boundaries.

- (a) Find the coordinates of A and of B .

[4 marks]



Answer $A(\dots\dots, \dots\dots)$ $B(\dots\dots, \dots\dots)$

- (b) Use inequalities to define the region R .

[3 marks]

Answer

- (c) Hence, or otherwise, find the set of values of x for which $2x^2 - 5x - 3 > x + 5$.

[2 marks]

Answer

Total for D1: 9 marks

D2

Priya solves $\frac{10}{x} > x - 3$, $x \neq 0$. She writes: "I multiply both sides by x to get $10 > x^2 - 3x$. So $x^2 - 3x - 10 < 0$, $(x - 5)(x + 2) < 0$, and the answer is $-2 < x < 5$."

Priya is wrong.

- (a) Explain the mistake in Priya's method, and give a value of x in her answer that does not satisfy $\frac{10}{x} > x - 3$. [2 marks]

- (b) Find, in set notation, the correct set of values of x for which $\frac{10}{x} > x - 3$. [4 marks]

Answer

Total for D2: 6 marks

- D3** The curve C has equation $y = x^2 + 4$. The line l has equation $y = mx$, where m is a positive constant. The line l is a tangent to C at the point P .

- (a) Find the value of m . [3 marks]

Answer $m = \dots\dots$

(b) Hence find the coordinates of P .

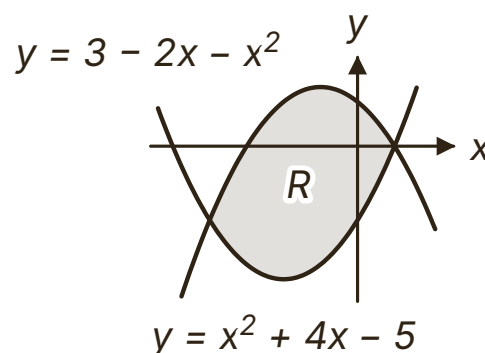
[2 marks]

Answer $P(\dots\dots, \dots\dots)$

Total for D3: 5 marks

D4 The shaded region R in the sketch is the set of points with $y \leq 3 - 2x - x^2$ and $y \geq x^2 + 4x - 5$.

(a) Find the coordinates of the points where the two curves meet. [4 marks]



Answer

(b) Hence find, in set notation, the set of values of x for which $3 - 2x - x^2 \geq x^2 + 4x - 5$.

[2 marks]

Answer

(c) Find the greatest value and the least value of y for points in R . [3 marks]

Answer greatest least

Total for D4: 9 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 How many points with integer coordinates (x, y) satisfy both $y \geq x^2 - 9$ and $y < 9 - x^2$? [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Take one value of x at a time. For which whole numbers x can there be any y at all?
- Hint 2** For a fixed whole number x , y runs from $x^2 - 9$ up to $8 - x^2$, so there are $(8 - x^2) - (x^2 - 9) + 1 = 18 - 2x^2$ values.
- Hint 3** $18 - 2x^2 > 0$ needs $x^2 < 9$, so $x = -2, -1, 0, 1, 2$. That gives $10 + 16 + 18 + 16 + 10 = 70$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can read the solution of an inequality from two graphs. ☐ ☐ ☐

I can handle a reciprocal graph, where $x = 0$ is a critical value. ☐ ☐ ☐

I can shade a region using dashed and solid boundaries. ☐ ☐ ☐

I can write down the inequalities that define a shaded region. ☐ ☐ ☐