Integrating standard functions and f(ax + b)

114 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-integrating-standard-functions/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Find $\frac{d}{dx}(\tan 4x)$.

Answer .....

A2 Needed today [2 marks]

Find $\int \left(6x^2 - \frac{4}{x^3}\right) dx$.

Answer .....

A3 Common slip [2 marks]

Differentiate $\ln 5x$.

Answer .....

A4 Needed today [3 marks]

Write $\frac{3}{\sqrt{x}}$ as a power of x, then find $\int \frac{3}{\sqrt{x}}\, dx$.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find $\int \left(3e^x - \frac{2}{x} + \cos x\right) dx.$

Hint Each term is a standard result: $\int e^x \, dx = e^x$, $\int \frac{1}{x} \, dx = \ln|x|$, $\int \cos x \, dx = \sin x$.

B1 Find $\int (5\sec^2 x + \frac{3}{x} - 2\sin x) \, dx.$ [4 marks]

Answer .....

EXAMPLE 2

Find

$\int (\sec x \tan x - 4\,\mathrm{cosec}\, x \cot x) \, dx.$

Hint Read the derivatives backwards. The derivative of $\sec x$ is $\sec x \tan x$, and the derivative of $\mathrm{cosec}\, x$ is $-\mathrm{cosec}\, x \cot x$.

B2 Find $\int (2\,\mathrm{cosec}^2 x - 3\sec x \tan x) \, dx.$ [4 marks]

Answer .....

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EXAMPLE 3

Find $\int \left(e^{3x+2} + \cos(4x-1)\right) dx.$

Hint Integrate as if the bracket were x, then divide by the number in front of x. Check by differentiating: the chain rule multiplies by that number.

B3 Find $\int \left(\sin(5x+2) + e^{1-2x}\right) dx.$ [5 marks]

Answer .....

EXAMPLE 4

Find $\int \dfrac{4}{2x+3}\, dx.$

Hint $\int \dfrac{1}{ax+b}\, dx = \dfrac{1}{a}\ln|ax+b|$. Do not forget to divide by a.

B4 Find $\int \left(\frac{6}{3x-1} + \frac{1}{(2x+1)^3}\right) dx.$ [5 marks]

Answer .....

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EXAMPLE 5

Find the exact value of

$$\int_0^{\frac{\pi}{6}} \sec^2 2x \, dx.$$

Hint Integrate, then put in the limits. Use exact values: $\tan \frac{\pi}{3} = \sqrt{3}$.

B5 Find the exact value of $\int_1^4 \frac{3}{2x+1} \, dx$, giving your answer in the form $k \ln 3$. [5 marks]

Answer .....

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C1 Jo writes: "$\int e^{4x}\,dx = 4e^{4x} + c$." Explain her mistake. Then find the exact value of $\int_0^{\frac{1}{2}} e^{4x}\,dx$. [4 marks]

Answer .....

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C2 [4 marks]

Sam writes: "$\int \frac{1}{3x+2}\,dx = \ln|3x+2| + c$." Show, by differentiating his answer, that he is wrong. Then find the exact value of $\int_0^2 \frac{1}{3x+2}\,dx$ in the form $k\ln 2$.

Answer .....

C3 [4 marks]

Find the value of $\int_0^1 (2x+1)^4\,dx$. A student gets 48.4. Say what the student has done.

Answer .....

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C4 Find $\int \frac{(x+2)^2}{x}\,dx$, $x > 0$. [4 marks]

Answer .....

C5 Given that $a > 1$ and $\int_1^a \frac{4}{2x-1}\,dx = \ln 25$, find the value of a. [5 marks]

Answer .....

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D1 The curve C has equation $y = f(x)$, $x > 0$, where $f'(x) = 6e^{3x} - \dfrac{4}{x}$. The point $\left(\dfrac{1}{3},\ 2e\right)$ lies on C.

(a) Show that $f(x) = 2e^{3x} - 4\ln 3x$. [5 marks]

(b) Find the exact value of $f(1)$. [2 marks]

Answer .....

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(c) Hence find the exact value of $\int_{\frac{1}{3}}^{1} f'(x)\,dx$. [2 marks]

Answer .....

Total for D1: 9 marks

D2

> Lily writes: "$\int \sin 3x\,dx = 3\cos 3x + c$, so $\int_0^{\frac{\pi}{6}} \sin 3x\,dx = 3\cos\frac{\pi}{2} - 3\cos 0 = -3$."

Lily is wrong.

(a) Explain the two mistakes Lily has made. [2 marks]

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(b) Find the correct value of $\int_0^{\frac{\pi}{6}} \sin 3x \, dx$. [3 marks]

Answer .....

(c) Hence find the exact value of $\int_0^{\frac{\pi}{6}} \left(2 \sin 3x + 3 \sec^2 2x\right) dx$. [4 marks]

Answer .....

Total for D2: 9 marks

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D3 Find the exact value of $\int_1^2 \left(\frac{4}{2x-1} + e^{2-x} \right) dx$. Give your answer in the form $a \ln 3 + e + b$, where a and b are integers. [7 marks]

Answer .....

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D4 This question is about $\displaystyle\int_0^k \frac{a}{(3x+1)^2}\,dx$, where a and k are positive constants.

(a) Show that $\displaystyle\int_0^1 \frac{a}{(3x+1)^2}\,dx = \frac{a}{4}$. [4 marks]

(b) Given that $a = 12$ and $\displaystyle\int_0^k \frac{12}{(3x+1)^2}\,dx = 3$, find k. [3 marks]

Answer .....

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(c) Explain why $\displaystyle\int_0^k \frac{12}{(3x+1)^2}\,dx < 4$ for every $k > 0$. [2 marks]

Total for D4: 9 marks

D5 This question is about $\operatorname{cosec} 2x$.

(a) Show that $\dfrac{d}{dx}(\operatorname{cosec} 2x) = -2\operatorname{cosec} 2x \cot 2x$. [2 marks]

(b) Hence find $\displaystyle\int \operatorname{cosec} 2x \cot 2x\,dx$. [2 marks]

Answer .....

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(c) Find the exact value of $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} \left(\operatorname{cosec} 2x \cot 2x + \operatorname{cosec}^2 2x\right) dx.$ [4 marks]

Answer .....

Total for D5: 8 marks

D6 This question is about $\left(e^x + e^{-x}\right)^2$.

(a) Find $\int \left(e^x + e^{-x}\right)^2 dx.$ [3 marks]

Answer .....

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(b) Hence show that $\int_0^{\ln 2} \left(e^x + e^{-x}\right)^2 dx = \frac{15}{8} + 2\ln 2.$ [4 marks]

(c) Given that $\int_0^k e^{2x}\, dx = 4$, find the exact value of k. [3 marks]

Answer .....

Total for D6: 10 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 [9 marks]

For each integer $n \geq 0$, let $I_n = \displaystyle\int_n^{n+1} \frac{4}{2x+1}\,dx$. Find the least integer n for which $I_n < 0.1$. Then find, in the form $a \ln 3$, the exact value of $I_0 + I_1 + I_2 + \ldots + I_{39}$.

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 $I_n = [2\ln(2x+1)]_n^{n+1} = 2\ln\left(\dfrac{2n+3}{2n+1}\right)$.

Hint 2 $I_n < 0.1$ means $\dfrac{2n+3}{2n+1} < e^{0.05}$, which is $1 + \dfrac{2}{2n+1} < e^{0.05}$. Solve for n and test the integers either side.

Hint 3 The sum of the integrals over $[0,1], [1,2], \ldots, [39,40]$ is one integral from 0 to 40.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

- I can integrate e^x, 1/x and the six standard trigonometric results. ○ ○ ○
- I can integrate f(ax + b) by integrating f and dividing by a. ○ ○ ○
- I can write ln|x| with the modulus and use log laws to give an exact answer. ○ ○ ○
- I can find an unknown limit or constant from the value of a definite integral. ○ ○ ○

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