

Integrating x^n and finding functions

71 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-integrating-x-and-finding-functions/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] Write $\frac{4}{x^3} - 3\sqrt{x}$ in the form $ax^p + bx^q$.
- A2** Last lesson [2 marks] $V = 48x - x^3$. Find the positive value of x for which V is stationary.

Answer

Answer $x = \dots\dots$

- A3** From chapter 12 [2 marks] Find $\frac{dy}{dx}$ when $y = 6x^{\frac{4}{3}} + \frac{2}{x}$.
- A4** From chapter 1 [2 marks] Expand and simplify $(3\sqrt{x} - 2)^2$.

Answer

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find $\int (8x^3 - 6x^{-4} + 3x^{\frac{1}{2}}) dx$.

Hint Add one to each power, then divide by the new power. Do each term on its own, and add c once at the end.

B1 Find [3 marks]

$\int (10x^4 + 9x^{-4} - 2x^{-\frac{1}{2}}) dx$.

Answer

EXAMPLE 2

Find $\int \frac{x^2 - 3}{2\sqrt{x}} dx$.

Hint You cannot integrate the top and the bottom separately. Divide each term on top by $2x^{\frac{1}{2}}$ first.

B2 Find [4 marks]

$\int \frac{(2x + 3)(x - 4)}{\sqrt{x}} dx$,
giving each term in its simplest form.

Answer

EXAMPLE 3

A curve has $\frac{dy}{dx} = 3x^2 + kx - 4$, where k is a constant. The curve has a stationary point where $x = 2$, and it passes through $(1, 5)$. Find the equation of the curve.

Hint At a stationary point $\frac{dy}{dx} = 0$: that gives k first. Then integrate and use $(1, 5)$ to find c .

B3

A curve has

[5 marks]

$\frac{dy}{dx} = k\sqrt{x} - 3x$, where k is a constant, and $x > 0$. The curve has a stationary point at $(4, 10)$. Find k and the equation of the curve.

Answer

c Practice

A-Level

- C1** Given that $\int \left(p\sqrt{x} - \frac{3q}{x^2} \right) dx = 4x^{\frac{3}{2}} + \frac{12}{x} + c$, where p and q are constants, find p and q . [3 marks]

Answer $p = \dots\dots$ $q = \dots\dots$

C2 Given that $y^{\frac{1}{2}} = 2x^{\frac{1}{4}} - x^{-\frac{1}{4}}$, $x > 0$, find $\int y \, dx$. [4 marks]

Answer

C3 $f''(x) = 12x - 4$. The curve $y = f(x)$ passes through $(1, 2)$, and its gradient at that point is 3. Find $f(x)$. [4 marks]

Answer $f(x) = \dots\dots$

D Exam-style questions

Edexcel exam style

D1 $f'(x) = \frac{(x^2 - 3)^2}{x^2}$, $x \neq 0$

(a) Show that $f'(x) = x^2 - 6 + 9x^{-2}$. [2 marks]

(b) Given that the point $(3, -4)$ lies on the curve $y = f(x)$, find $f(x)$. [4 marks]

Answer $f(x) = \dots\dots$

(c) Find $f''(x)$. Hence show that the curve $y = f(x)$ has a stationary point where $x = \sqrt{3}$, and that $f''(\sqrt{3}) = 0$. [4 marks]

Total for D1: 10 marks

D2

Leo writes: $\int \frac{6x^3 - 5}{x^2} dx = \frac{\frac{3}{2}x^4 - 5x}{\frac{1}{3}x^3} + c$

Leo is wrong.

(a) Explain the mistake Leo has made, and find $\int \frac{6x^3 - 5}{x^2} dx$ correctly, giving each term in its simplest form. [4 marks]

Answer

- (b) A curve has gradient $\frac{6x^3 - 5}{x^2}$ and passes through $(1, 2)$. Hence [2 marks]
find the equation of the curve.

Answer $y = \dots\dots$

Total for D2: 6 marks

- D3** The volume, V cubic metres, of water in a tank t hours after a pump is switched on is modelled by

$$\frac{dV}{dt} = 24\sqrt{t} - 3t$$

When $t = 0$, $V = 50$. The model applies while the volume is increasing.

- (a) Find V in terms of t . [4 marks]

Answer $V = \dots\dots$

- (b) Find the time at which the model stops applying. Hence find the [4 marks]
greatest volume of water in the tank according to the model.

Answer

Total for D3: 8 marks

D4 The curve C has equation $y = f(x)$, where $f'(x) = 6x^2 + kx - 12$ and k is a constant. C passes through the points $(0, 5)$ and $(2, -15)$.

(a) Find the value of k and an expression for $f(x)$. [5 marks]

Answer

(b) Find the coordinates of the stationary points of C , and determine their nature. [5 marks]

Answer

Total for D4: 10 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The polynomial $f(x)$ satisfies

[6 marks]

$$f'(x) = 6x^2 - 2f(1)x + f(2)$$

for every x , and $f(0) = 1$. Find $f(x)$.

Answer

Extension hints

Cover these with paper. Uncover one at a time.

E1 Hint 1 $f(1)$ and $f(2)$ are just numbers. Call them a and b , and integrate.

Hint 2 $f(x) = 2x^3 - ax^2 + bx + 1$. Now put $x = 1$ and $x = 2$ into this, remembering that $f(1) = a$ and $f(2) = b$.

Hint 3 $a = 2 - a + b + 1$ and $b = 16 - 4a + 2b + 1$, so $2a - b = 3$ and $4a - b = 17$. Then $a = 7$ and $b = 11$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can integrate powers of x , including negative and fractional powers.



I can rewrite an expression in index form before integrating.



I can find a curve or a model from its rate of change and one point.

