# Integration by parts

114 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-integration-by-parts/

Date

## A Do Now

Quick questions to start.

**A1** Needed today [2 marks]

Differentiate $xe^{2x}$ with respect to $x$.

Answer .....

**A2** Needed today [2 marks]

Find $\int e^{3x}\,dx$.

Answer .....

**A3** Needed today [2 marks]

Differentiate $x \ln x$ with respect to $x$.

Answer .....

**A4** Common slip [2 marks]

Find $\int \cos 2x\,dx$. Check your answer by differentiating it.

Answer .....



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## B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

**EXAMPLE 1**

Find $\int xe^{x}\,dx$.

**Hint** Take $u = x$, because it becomes 1 when you differentiate it, and $\frac{dv}{dx} = e^{x}$, so $v = e^{x}$.

**B1** Find $\int xe^{3x}\,dx$. [4 marks]

Answer .....

**EXAMPLE 2**

Find $\int x\cos x\,dx$.

**Hint** Take $u = x$ and $\frac{dv}{dx} = \cos x$, so $v = \sin x$. Watch the sign when you integrate $\sin x$.

**B2** Find $\int x\sin 2x\,dx$. [4 marks]

Answer .....



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**EXAMPLE 3**

Find $\int \ln x \, dx$.

**Hint** Write it as $\int 1 \times \ln x \, dx$. Take $u = \ln x$, because you cannot integrate it yet, and $\frac{dv}{dx} = 1$, so $v = x$.

**B3** Find the exact value of $\int_1^e x^2 \ln x \, dx$. [5 marks]

Answer .....



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**EXAMPLE 4**

Find $\int x^2 e^x \, dx$.

**Hint** Use parts with $u = x^2$. The new integral $\int 2x e^x \, dx$ needs parts again.

**B4** Find $\int x^2 \cos x \, dx$. [5 marks]

Answer .....



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**EXAMPLE 5**

Find $\int e^x \sin x \, dx$.

**Hint** Call it $I$. Use parts twice with $u$ as the trig function each time. The second time, $I$ appears again: collect it on one side.

**B5** Find the exact value of $\int_0^{\frac{\pi}{2}} e^x \cos x \, dx$. [6 marks]

Answer .....


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**C1** [5 marks]

Kai finds $\int_0^1 xe^{2x}\,dx$ by parts. He takes $u = e^{2x}$ and $\frac{dv}{dx} = x$, and gets $\frac{1}{2}x^2e^{2x} - \int x^2e^{2x}\,dx$. Explain why this does not help. Then find the exact value.

Answer .....


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**C2** Ella writes $\int x \sin x \, dx = -x \cos x - \int \cos x \, dx = -x \cos x - \sin x + c$. Find her mistake, then write the correct answer. [4 marks]

Answer .....

**C3** Find the exact value of $\int_1^2 \ln 3x \, dx$. Give your answer in the form $\ln a - b$. [5 marks]

Answer .....


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**C4** Find $\int x \sec^2 x \, dx$. [4 marks]

Answer .....

**C5** Use integration by parts to find the exact value of [4 marks]

$$\int_0^1 x \left(x+1\right)^3 \, dx.$$

Answer .....


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**D1** This question is about $\int xe^{-2x}\,dx$.

**(a)** Find $\int xe^{-2x}\,dx$. [4 marks]

Answer .....

**(b)** Find the exact value of $\int_0^{\ln 2} xe^{-2x}\,dx$. [3 marks]

Answer .....


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**(c)** Show that $\int_0^k xe^{-2x}\,dx < \frac{1}{4}$ for every $k > 0$. [2 marks]

**Total for D1: 9 marks**

**D2**

Sam writes $\int x^2e^{2x}\,dx = \frac{1}{2}x^2e^{2x} + \int xe^{2x}\,dx$.

Sam is wrong.

**(a)** Explain his mistake. [2 marks]


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**(b)** Find $\int x^2 e^{2x}\, dx$. [4 marks]

Answer .....

**(c)** Hence find the exact value of $\int_0^1 x^2 e^{2x}\, dx$. [3 marks]

Answer .....

**Total for D2: 9 marks**


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**D3** Find the exact value of $\displaystyle\int_1^e \frac{\ln x}{x^2}\,dx$. [7 marks]

Answer .....


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**D4** This question is about $I = \int e^{2x} \sin x \, dx$.

**(a)** Use integration by parts twice to show that $I = \frac{1}{5}e^{2x}(2\sin x - \cos x) + c$. [6 marks]


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**(b)** Find the exact value of $\int_0^{\pi} e^{2x} \sin x \, dx$. [3 marks]

Answer .....

**Total for D4: 9 marks**

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**D5** This question is about $\int_1^4 e^{\sqrt{x}} \, dx$.

**(a)** Use the substitution $u = \sqrt{x}$ to show that $\int_1^4 e^{\sqrt{x}} \, dx = \int_1^2 2ue^u \, du$. [3 marks]


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**(b)** Hence find the exact value of $\int_1^4 e^{\sqrt{x}}\,dx$. [6 marks]

Answer .....

**Total for D5: 9 marks**

**D6** This question is about $\int x \ln x\,dx$.

**(a)** Find $\int x \ln x\,dx$. [3 marks]

Answer .....



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**(b)** Find the exact value of $\int_1^3 x \ln x \, dx$. [2 marks]

Answer .....

**(c)** Use your answer to part (a) to find $\int x (\ln x)^2 \, dx$. [3 marks]

Answer .....

**Total for D6: 8 marks**


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## E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

**E1** Find the exact value of $\int_1^e (\ln x)^3 \, dx$. [9 marks]

Answer .....



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## Extension hints

Cover these with paper. Uncover one at a time.

**E1**

**Hint 1** Let $I_n = \int_1^e (\ln x)^n \, dx$. Use parts with $u = (\ln x)^n$ and $v = x$.

**Hint 2** You should get $I_n = e - nI_{n-1}$, because $\ln e = 1$ and $\ln 1 = 0$.

**Hint 3** Start from $I_0 = \int_1^e 1 \, dx = e - 1$ and work up to $I_3$.

## How did it go?

Colour one circle on each line: not yet, nearly, yes.

| | |
|---|---|
| I can choose u so that it gets simpler when I differentiate it, and use the parts formula with the minus sign. | ○ ○ ○ |
| I can integrate ln x, x² ln x and similar by taking u = ln x. | ○ ○ ○ |
| I can use integration by parts twice, and spot when the integral comes back to the start. | ○ ○ ○ |
| I can find an exact value with integration by parts and the limits. | ○ ○ ○ |



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