Integration with partial fractions

116 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-integration-partial-fractions/

Date

A Do Now

Quick questions to start.

A1 Needed today [3 marks]

Write $\dfrac{x+7}{(x-1)(x+3)}$ in partial fractions.

Answer .....

A2 Needed today [2 marks]

Find $\displaystyle\int \frac{3}{x+2}\,dx$.

Answer .....

A3 Common slip [2 marks]

Find $\displaystyle\int \frac{4}{2x-1}\,dx$.

Check your answer by differentiating it.

Answer .....

A4 Needed today [2 marks]

Write $2\ln 3 - \ln 6$ as a single logarithm.

Answer .....

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B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find $\displaystyle\int \frac{5x+1}{(x-1)(x+2)}\,dx.$

Hint Split it first: it is $\dfrac{2}{x-1}+\dfrac{3}{x+2}$.
Each part integrates to a logarithm.

B1 Find [4 marks]

$$\int \frac{4x+5}{(x+2)(x+1)}\,dx.$$

Answer .....

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EXAMPLE 2

Find the exact value of

$$\int_2^3 \frac{3}{(x-1)(x+2)}\,dx.$$

Hint It is $\frac{1}{x-1} - \frac{1}{x+2}$. Integrate, put in the limits, then use the log laws to make one logarithm.

B2 Find the exact value of $\int_3^4 \frac{5}{(2x+1)(x-2)}\,dx$. Give your answer as a single logarithm. [6 marks]

Answer .....

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EXAMPLE 3

Find $\displaystyle\int \frac{x+3}{(x+1)^2}\,dx.$

Hint Write it as $\dfrac{A}{x+1}+\dfrac{B}{(x+1)^2}$. The second part is a power: $\displaystyle\int (x+1)^{-2}\,dx = -(x+1)^{-1}$, not a logarithm.

B3 Find $\displaystyle\int \frac{3x+1}{(x+1)^2}\,dx.$ [4 marks]

Answer .....

EXAMPLE 4

Find $\displaystyle\int \frac{x^2}{x^2-1}\,dx.$

Hint The top has the same degree as the bottom, so divide first: $\dfrac{x^2}{x^2-1} = 1 + \dfrac{1}{x^2-1}$. Then split $\dfrac{1}{(x-1)(x+1)}$.

B4 Find the exact value of $\displaystyle\int_2^3 \frac{x^2+1}{x^2-1}\,dx.$ [6 marks]

Answer .....

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EXAMPLE 5

Find $\int \frac{1}{x(x+1)}\,dx$ and write the answer as a single logarithm.

Hint $\frac{1}{x(x+1)} = \frac{1}{x} - \frac{1}{x+1}$. A difference of logarithms is the logarithm of a quotient.

B5 Find the exact value of $\int_1^4 \frac{2}{x(x+2)}\,dx$. [4 marks]

Answer .....

C Practice

A-Level

C1 Nia writes $\int_0^4 \frac{3}{2x+1}\,dx = \left[3\ln|2x+1|\right]_0^4 = 3\ln 9$. Find her mistake, then find the correct value in the form $k\ln 3$. [4 marks]

Answer .....

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C2 Max tries to split $\dfrac{x^2+x}{(x+2)(x-1)}$ as $\dfrac{A}{x+2}+\dfrac{B}{x-1}$ and cannot make it work. Explain why, then find $\displaystyle\int \frac{x^2+x}{(x+2)(x-1)}\,dx$. [5 marks]

Answer .....

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C3 Find the exact value of $\int_2^3 \frac{1}{x(x-1)}\,dx$. Give your answer as a single logarithm. [4 marks]

Answer .....

C4 Find $\int \frac{8}{x^2-16}\,dx$, writing your answer as a single logarithm. [4 marks]

Answer .....

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C5 Find the exact value of $\displaystyle\int_0^1 \frac{x}{(x+1)^2}\,dx$. [5 marks]

Answer .....

D Exam-style questions

Edexcel exam style

D1 $\mathrm{f}(x) = \dfrac{x+10}{(2x-1)(x+3)}$.

(a) Write $\mathrm{f}(x)$ in partial fractions. [3 marks]

Answer .....

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(b) Find the exact value of $\int_1^2 \mathrm{f}(x)\,dx$, giving your answer in the form $\ln k$. [5 marks]

Answer .....

(c) Explain why $\int_0^1 \mathrm{f}(x)\,dx$ cannot be found with this method. [2 marks]

Total for D1: 10 marks

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D2

Raj writes $\int_1^3 \frac{4}{(2x+1)(2x+3)}\,dx = \left[2\ln|2x+1| - 2\ln|2x+3|\right]_1^3$.

Raj is wrong.

(a) Explain his mistake. [2 marks]

(b) Find the exact value of $\int_1^3 \frac{4}{(2x+1)(2x+3)}\,dx$, giving your answer as a single logarithm. [4 marks]

Answer .....

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(c) Show that $\displaystyle\int_1^n \frac{4}{(2x+1)(2x+3)}\,dx < \ln\frac{5}{3}$ for every $n > 1$. [3 marks]

Total for D2: 9 marks

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D3 Find the exact value of $\displaystyle\int_3^5 \frac{2x^2-3}{x^2-4}\,dx$, giving your answer in the form $a + b\ln c$, where a, b and c are rational. [7 marks]

Answer .....

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D4 $\mathrm{g}(x) = \dfrac{2x^2 + 3x + 4}{x^2(x + 2)}$.

(a) Write $\mathrm{g}(x)$ in the form $\dfrac{A}{x} + \dfrac{B}{x^2} + \dfrac{C}{x + 2}$. [4 marks]

Answer .....

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(b) Find the exact value of $\int_1^2 \mathrm{g}(x)\,dx$, giving your answer in the form $p + q\ln 2 + r\ln 3$. [5 marks]

Answer .....

Total for D4: 9 marks

D5 This question is about $\int_0^{\ln 2} \frac{e^x}{(e^x+1)(e^x+2)}\,dx$.

(a) Use the substitution $u = e^x$ to show that the integral is $\int_1^2 \frac{1}{(u+1)(u+2)}\,du$. [2 marks]

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(b) Hence find the exact value of the integral, giving your answer as a single logarithm. [6 marks]

Answer .....

Total for D5: 8 marks

D6 $\mathrm{h}(x) = \dfrac{3}{(1-x)(1+2x)}$.

(a) Write $\mathrm{h}(x)$ in partial fractions. [3 marks]

Answer .....

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(b) Find the exact value of $\int_0^{\frac{1}{2}} \mathrm{h}(x)\,dx$. [4 marks]

Answer .....

(c) $\int_0^k \mathrm{h}(x)\,dx$, with $k > 0$, has a finite value. State the values of k for which this is true, and give a reason. [2 marks]

Total for D6: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 $\displaystyle\int_2^k \frac{4}{x^2-1}\,dx = \ln 2$, where $k > 2$. Find the exact value of k, in the form $\dfrac{a+b\sqrt{2}}{c}$. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Split $\dfrac{4}{x^2-1}$ into $\dfrac{2}{x-1}-\dfrac{2}{x+1}$ and integrate.

Hint 2 The integral is $2\ln\dfrac{k-1}{k+1}-2\ln\dfrac{1}{3}=2\ln\dfrac{3(k-1)}{k+1}$. Set this equal to $\ln 2$.

Hint 3 So $\dfrac{3(k-1)}{k+1}=\sqrt{2}$. Solve for k and rationalise the denominator.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can split a fraction into partial fractions and integrate each part to a logarithm. ○ ○ ○

I can integrate $\dfrac{k}{ax+b}$ to $\dfrac{k}{a}\ln|ax+b|$ without losing the $\dfrac{1}{a}$. ○ ○ ○

I can integrate a repeated factor, where $\dfrac{1}{(x+a)^2}$ does not give a logarithm. ○ ○ ○

I can divide an improper fraction first, and write an exact answer as a single logarithm. ○ ○ ○

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