Trigonometric identities and the reverse chain rule

113 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-integration-trig-identities-reverse-chain-rule/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Use $\cos 2x = 1 - 2\sin^2 x$ to write $\sin^2 x$ in terms of $\cos 2x$.

Answer .....

A2 Needed today [2 marks]

Differentiate $\ln\left(x^2 + 5\right)$.

Answer .....

A3 Needed today [2 marks]

Differentiate $\left(x^3 + 1\right)^5$.

Answer .....

A4 Common slip [3 marks]

Write $\tan^2 x$ in terms of $\sec^2 x$. Then find $\int \tan^2 x \, dx$.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find $\int \cos^2 x \, dx$.

Hint There is no rule for a square of $\cos x$. Use $\cos 2x = 2\cos^2 x - 1$ to write $\cos^2 x = \frac{1}{2} + \frac{1}{2}\cos 2x$, then integrate each term.

B1 Find $\int \sin^2 3x \, dx$. [5 marks]

Answer .....

EXAMPLE 2

Find $\int \cot^2 x \, dx$.

Hint Divide $\sin^2 x + \cos^2 x = 1$ by $\sin^2 x$: $1 + \cot^2 x = \operatorname{cosec}^2 x$. And $\int \operatorname{cosec}^2 x \, dx = -\cot x$.

B2 Find $\int \cot^2 2x \, dx$. [4 marks]

Answer .....

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EXAMPLE 3

Find $\int \frac{6x^2}{x^3+2}\,dx$.

Hint The derivative of the bottom, x^3+2, is $3x^2$. The top is $2\times 3x^2$, so the integral is $2\ln|x^3+2|$. Check by differentiating.

EXAMPLE 4

Find $\int x\left(x^2+1\right)^3\,dx$.

Hint Try $\left(x^2+1\right)^4$. Its derivative is $4\left(x^2+1\right)^3\times 2x = 8x\left(x^2+1\right)^3$, which is 8 times too big.

B3 Find the exact value of $\int_0^{\frac{\pi}{2}} \frac{\cos x}{3+2\sin x}\,dx$. [5 marks]

Answer .....

B4 Find $\int \sin x\cos^4 x\,dx$. [4 marks]

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Answer .....

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EXAMPLE 5

Find $\int \tan x \, dx$.

Hint Write $\tan x = \dfrac{\sin x}{\cos x}$. The top is minus the derivative of the bottom.

B5 Find the exact value of $\int_0^{\frac{\pi}{6}} \tan 2x \, dx$. [4 marks]

Answer .....

C Practice

A-Level

C1 Raj writes: "$\int \sin^2 x \, dx = \dfrac{1}{3}\sin^3 x + c$." Explain his mistake. Then find $\int \sin^2 x \, dx$. [4 marks]

Answer .....

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C2 Mia writes: "$\displaystyle\int \frac{1}{x^2+4}\,dx = \ln\left(x^2+4\right) + c$." Show, by differentiating her answer, that she is wrong. Then find $\displaystyle\int \frac{x}{x^2+4}\,dx$. [5 marks]

Answer .....

C3 Find the exact value of $\displaystyle\int_0^{\frac{\pi}{4}} \sin 2x \cos 2x\,dx$. [4 marks]

Answer .....

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C4 Find the exact value of $\displaystyle\int_0^{\ln 2} \frac{e^{2x}}{1+e^{2x}}\,dx$. Give your answer as a single logarithm. [5 marks]

Answer .....

C5 Find $\displaystyle\int (\sin x + \cos x)^2\,dx$. [4 marks]

Answer .....

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D1 This question is about $\cos^2 2x$.

(a) Show that $\cos^2 2x = \frac{1}{2} + \frac{1}{2}\cos 4x$. [2 marks]

(b) Hence find $\int \cos^2 2x \, dx$. [2 marks]

Answer .....

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(c) Find the exact value of $\int_0^{\frac{\pi}{8}} (\cos 2x + \sin 2x)^2 \, dx$. [5 marks]

Answer .....

Total for D1: 9 marks

D2

> Tom writes: "The derivative of $x^2 + 1$ is in the top, so $\int \frac{4x}{x^2+1} \, dx = 4 \ln (x^2 + 1) + c$."

Tom is wrong.

(a) Explain his mistake. [2 marks]

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(b) Find the exact value of $\displaystyle\int_0^2 \frac{4x}{x^2+1}\,dx$. [3 marks]

Answer .....

(c) Find the exact value of $\displaystyle\int_1^3 \frac{4x^3}{x^4+1}\,dx$, giving your answer in the form $\ln k$. [4 marks]

Answer .....

Total for D2: 9 marks

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D3 [8 marks]

Find the exact value of $\int_0^{\frac{\pi}{3}} \left(2\tan^2 x + \sin^2 x\right) dx$. Give your answer in the form $a\sqrt{3} + b\pi$, where a and b are rational.

Answer .....

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D4 This question is about $e^{\cos 2x}$.

(a) Find $\dfrac{d}{dx}\left(e^{\cos 2x}\right)$. [2 marks]

Answer .....

(b) Hence find $\displaystyle\int \sin x \cos x \, e^{\cos 2x} \, dx$. [3 marks]

Answer .....

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(c) Find the exact value of $\int_0^{\frac{\pi}{2}} \sin x \cos x \, e^{\cos 2x} \, dx$. [3 marks]

Answer .....

Total for D4: 8 marks

D5 This question is about $\tan x$ and $\cot x$ for $0 < x < \frac{\pi}{2}$.

(a) Show that $\int_0^{\frac{\pi}{4}} \tan x \, dx = \frac{1}{2} \ln 2$. [3 marks]

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(b) Find the exact value of $\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cot x \, dx$. [3 marks]

Answer .....

(c) Given that $\int_0^k \tan x \, dx = \ln 2$, where $0 < k < \frac{\pi}{2}$, find the exact value of k. [3 marks]

Answer .....

Total for D5: 9 marks

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D6 This question is about $\dfrac{x^2}{(x^3+1)^3}$.

(a) Find $\displaystyle\int \frac{x^2}{(x^3+1)^3}\,dx$. [3 marks]

Answer .....

(b) Find the value of $\displaystyle\int_0^1 \frac{x^2}{(x^3+1)^3}\,dx$. [2 marks]

Answer .....

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(c) Show that $\displaystyle\int_0^k \frac{x^2}{\left(x^3+1\right)^3}\,dx < \frac{1}{6}$ for every $k > 0$. [3 marks]

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Find the exact value of $\displaystyle\int_0^{\frac{\pi}{4}} \sin^4 x \, dx$. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write $\sin^4 x = \left(\sin^2 x\right)^2$ and use $\sin^2 x = \frac{1}{2} - \frac{1}{2}\cos 2x$.

Hint 2 Squaring gives a $\cos^2 2x$ term. Use the identity again: $\cos^2 2x = \frac{1}{2} + \frac{1}{2}\cos 4x$.

Hint 3 You should reach $\sin^4 x = \frac{3}{8} - \frac{1}{2}\cos 2x + \frac{1}{8}\cos 4x$. Integrate each term.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can use the double angle formulae to integrate sin²x, cos²x and sin x cos x.	○ ○ ○
I can use tan²x = sec²x − 1 and cot²x = cosec²x − 1 before integrating.	○ ○ ○
I can recognise f′(x)/f(x) and integrate it to ln|f(x)|.	○ ○ ○
I can recognise f′(x)[f(x)]ⁿ, integrate it, and check by differentiating.	○ ○ ○

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