Inverse functions

124 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-inverse-functions/

Date

A Do Now

Quick questions to start.

A1 Needed today [3 marks]

Make x the subject of $y = \dfrac{3x + 1}{x - 2}$.

Answer .....

A2 Last lesson [3 marks]

$f(x) = 2x^2 - 12x + 7$, $x \in \mathbb{R}$. Find the range of f.

Answer .....

A3 Last lesson [2 marks]

$f(x) = x^2 - 1$ and $g(x) = 3x + 2$, $x \in \mathbb{R}$. Find $gf(2)$.

Answer .....

A4 From Year 1 [2 marks]

Solve $\ln(2x - 1) = 3$, giving your answer in exact form.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

The function f is defined by $f(x) = 4x - 7$, $x \in \mathbb{R}$. Find $f^{-1}(x)$ and $f^{-1}(5)$.

Hint Write $y = f(x)$, make x the subject, then swap the letters.

B1 The function f is defined by $f(x) = 3x + 8$, $x \in \mathbb{R}$. Find $f^{-1}(x)$ and $f^{-1}(2)$. [3 marks]

Answer .....

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EXAMPLE 2

$f(x) = \dfrac{x+5}{x-1}$, $x > 1$. Find $f^{-1}(x)$ and state its domain.

Hint The domain of f^{-1} is the range of f. Write $f(x)$ as $1 + \dfrac{6}{x-1}$ to see the range.

B2 $f(x) = \dfrac{2x+3}{x-4}$, $x > 4$ [4 marks]

. Find $f^{-1}(x)$ and state its domain.

Answer .....

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EXAMPLE 3

$f(x) = (x-2)^2 + 3$, $x \geq 2$. Find $f^{-1}(x)$, and state the domain and range of f^{-1}.

Hint On $x \geq 2$ the bracket $x - 2$ is 0 or more, so take the positive square root.

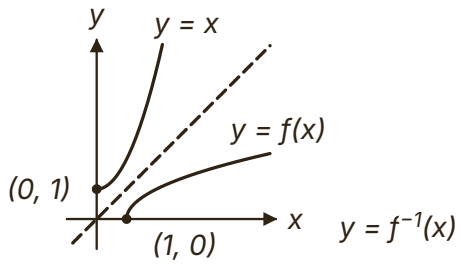
B3 $f(x) = x^2 + 6x + 1$, $x \geq -3$. Find $f^{-1}(x)$, and state the domain and range of f^{-1}. [5 marks]

Answer .....

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EXAMPLE 4

$f(x) = x^2 + 1$, $x \geq 0$. Find $f^{-1}(x)$ and sketch $y = f(x)$ and $y = f^{-1}(x)$ on the same axes.



Hint The graph of $y = f^{-1}(x)$ is the reflection of $y = f(x)$ in the line $y = x$.

B4 $f(x) = e^x + 2$, $x \in \mathbb{R}$. [4 marks]

(a) Find $f^{-1}(x)$ and state its domain.

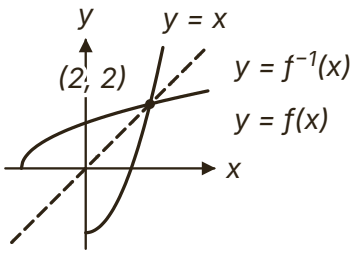
(b) Give the coordinates of the point where $y = f^{-1}(x)$ crosses the x-axis.

Answer .....

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EXAMPLE 5

$f(x) = x^2 - 2$, $x \geq 0$. Find the point where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ meet.



Hint f is increasing on its domain, so the two graphs meet on the line $y = x$. Solve $f(x) = x$.

B5 $f(x) = x^2 - 6$, $x \geq 0$. [4 marks]

Find the coordinates of the point where the graphs of $y = f(x)$ and $y = f^{-1}(x)$ meet.

Answer .....

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C1 $f(x) = 2x + 5$, $x \in \mathbb{R}$. Sam writes: "$f^{-1}(x) = \dfrac{1}{2x+5}$." [4 marks]

(a) Find the correct $f^{-1}(x)$.

(b) Show that Sam's function gives the wrong value of $f^{-1}(7)$.

Answer .....

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C2 For each function, state whether it has an inverse. Give a reason each time. [5 marks]

(a) $f(x) = x^2 - 4x$, $x \in \mathbb{R}$

(b) $g(x) = x^2 - 4x$, $x \geq 2$

(c) $h(x) = \dfrac{1}{x}$, $x \neq 0$

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C3 $f(x) = \dfrac{3x - 5}{x - 3}$, $x \in \mathbb{R}$, $x \neq 3$. Show that f is self-inverse, and hence write down $ff(7)$. [4 marks]

Answer .....

C4 $f(x) = x^2 + 4x$, $x \geq -2$. Leo says: "The domain of f is $x \geq -2$, so the domain of f^{-1} is also $x \geq -2$." [5 marks]

Explain why Leo is wrong. Find $f^{-1}(x)$ and its domain.

Answer .....

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C5 $f(x) = \ln(2x + 1)$, $x > -\frac{1}{2}$. [5 marks]

(a) Find $f^{-1}(x)$ and state its domain.

(b) Solve $f^{-1}(x) = 4$, giving your answer in the form $a \ln b$ where a and b are integers.

Answer .....

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C6 $f(x) = 2x - 1$ and $g(x) = x^3$, $x \in \mathbb{R}$. [5 marks]

(a) Find $(fg)^{-1}(x)$.

(b) Show that $(fg)^{-1}(x) = g^{-1}f^{-1}(x)$.

Answer .....

D Exam-style questions

Edexcel exam style

D1 The function f is defined by $f(x) = \dfrac{2x+1}{x-3}$, $x > 3$.

(a) Find the range of f. [2 marks]

Answer .....

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(b) Find $f^{-1}(x)$ and state its domain. [4 marks]

Answer .....

(c) Solve $f^{-1}(x) = 7$. [2 marks]

Answer .....

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(d) The function g is defined by $g(x) = x^2 + 2$, $x \geq 0$. Using your answer to part (b), or otherwise, find the exact value of a for which $f^{-1}g(a) = 4$. [3 marks]

Answer .....

Total for D1: 11 marks

D2

> $f(x) = x^2 - 8x + 10$, $x \geq 2$. Mia says: "The domain of f has been restricted, so f has an inverse."

Mia is wrong.

(a) Show that f does not have an inverse. [2 marks]

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(b) The domain is changed to $x \geq k$. Find the least value of k for which f has an inverse. [2 marks]

Answer .....

(c) With this value of k, find $f^{-1}(x)$ and state its domain. [4 marks]

Answer .....

Total for D2: 8 marks

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D3 The functions f and g are defined by $f(x) = x^2 - 8x + 21$, $x \geq 4$, and $g(x) = x^2 - 8x + 21$, $x \in \mathbb{R}$. Find $f^{-1}(x)$ and state its domain. Show that $f^{-1}g(x) = 4 + |x - 4|$. Hence state the set of values of x for which $f^{-1}g(x) = x$, and solve $f^{-1}g(x) = 2x$. [9 marks]

Answer .....

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D4 The functions f and g are defined by $f(x) = e^{2x} + 3$, $x \in \mathbb{R}$, and $g(x) = \ln(x - 3)$, $x > 3$.

(a) Find $f^{-1}(x)$ and state its domain. [3 marks]

Answer .....

(b) Show that $gf(x) = 2x$. [2 marks]

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(c) Find $fg(x)$ in its simplest form and solve $fg(x) = 19$. [4 marks]

Answer .....

Total for D4: 9 marks

D5 The function f is defined by $f(x) = \dfrac{4}{x+2}$, $x \geq 0$.

(a) Find the range of f. [2 marks]

Answer .....

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(b) Find $f^{-1}(x)$ and state its domain. [3 marks]

Answer .....

(c) Solve $f(x) = f^{-1}(x)$, giving your answer in surd form. [3 marks]

Answer .....

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(d) Show that $ff(x) = \dfrac{2(x+2)}{x+4}$. [3 marks]

Total for D5: 11 marks

D6 The function f is defined by $f(x) = \sqrt{2x+6}$, $x \geq -3$.

(a) Find $f^{-1}(x)$ and state its domain. [3 marks]

Answer .....

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(b) State the range of f^{-1}. [2 marks]

Answer .....

(c) On the same axes, sketch $y = f(x)$ and $y = f^{-1}(x)$. Give the coordinates of the points where each graph meets the coordinate axes. [4 marks]

Answer .....

Total for D6: 9 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 The function f is defined by $f(x) = x^2 - 2px + 3$, $x \geq p$, where p is a positive constant. Find the set of values of p for which the graphs of $y = f(x)$ and $y = f^{-1}(x)$ meet at exactly two points. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 f is increasing on $x \geq p$. So the two graphs can only meet on the line $y = x$. Solve $f(x) = x$.

Hint 2 $x^2 - (2p+1)x + 3 = 0$ needs two different roots, and both roots must be p or more, since the domain of f is $x \geq p$.

Hint 3 The larger root is always more than p. The smaller root is $\dfrac{2p+1-\sqrt{(2p+1)^2-12}}{2}$. Find when this is p or more.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can say whether a function has an inverse, and restrict its domain so that it does. ○ ○ ○

I can find $f^{-1}(x)$ by writing $y = f(x)$ and making x the subject. ○ ○ ○

I can state the domain and range of f^{-1} from the range and domain of f. ○ ○ ○

I can sketch $y = f^{-1}(x)$ as the reflection of $y = f(x)$ in $y = x$ and solve $f(x) = f^{-1}(x)$. ○ ○ ○

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