Inverse trigonometric functions

114 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-inverse-trig-functions/

Date

A Do Now

Quick questions to start.

A1 Needed today [3 marks]

$f(x) = x^2 + 1, x \geq 0$.
Find $f^{-1}(x)$ and state its domain.

Answer .....

A2 Needed today [3 marks]

Write down the exact values of $\sin \frac{\pi}{3}$, $\cos \frac{3\pi}{4}$ and $\tan\left(-\frac{\pi}{6}\right)$.

Answer .....

A3 Needed today [2 marks]

Solve $\cos x = -\frac{1}{2}$ for $0 \leq x \leq \pi$.

Answer .....

A4 Today's idea [2 marks]

Kai says that $\arcsin x$ means $\frac{1}{\sin x}$. Explain why Kai is wrong, and find $\arcsin \frac{1}{2}$.

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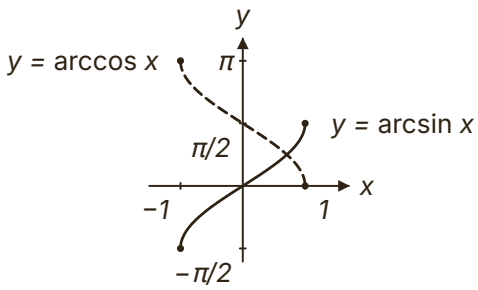
B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find the exact values of $\arccos\dfrac{1}{2}$, $\arcsin\left(-\dfrac{\sqrt{3}}{2}\right)$ and $\arctan(-1)$.



Hint arcsin and arctan give an answer between −π/2 and π/2. arccos gives an answer between 0 and π.

B1 Find the exact values of $\arcsin\left(-\dfrac{1}{2}\right)$, $\arccos\left(-\dfrac{\sqrt{2}}{2}\right)$, $\arctan\sqrt{3}$ and $\arccos 0$. [4 marks]

Answer .....

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EXAMPLE 2

Find the exact values of $\cos\left(\arcsin\frac{3}{5}\right)$ and $\tan\left(\arccos\left(-\frac{4}{5}\right)\right)$.

Hint Call the angle θ, draw a right-angled triangle for its size, then decide the signs from the range of the inverse function.

B2 [4 marks]

Find the exact values of $\sin\left(\arccos\frac{8}{17}\right)$ and $\tan\left(\arcsin\left(-\frac{5}{13}\right)\right)$.

Answer .....

EXAMPLE 3

Show that $\cos(\arcsin x) = \sqrt{1-x^2}$ for $-1 \le x \le 1$.

Hint Let y = arcsin x. Use sin² y + cos² y = 1, then decide the sign of cos y from the range of arcsin.

B3 [4 marks]

Let $y = \arctan x$. Show that $\cos y = \frac{1}{\sqrt{1+x^2}}$, and hence that $\sin y = \frac{x}{\sqrt{1+x^2}}$.

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EXAMPLE 4

Find the exact values of $\arcsin\left(\sin\frac{5\pi}{6}\right)$ and $\arccos\left(\cos\left(-\frac{\pi}{3}\right)\right)$.

Hint The answer must lie in the range of the inverse function, so it is not always the angle you started with.

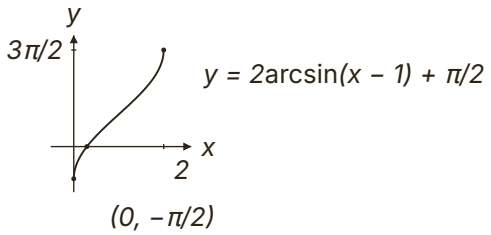
B4 Find the exact values of $\arcsin\left(\sin\left(-\frac{\pi}{3}\right)\right)$, $\arccos\left(\cos\frac{5\pi}{4}\right)$ and $\arctan\left(\tan\frac{3\pi}{4}\right)$. [3 marks]

Answer .....

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EXAMPLE 5

The curve C has equation $y = 2\arcsin(x-1) + \frac{\pi}{2}$. Find the domain and range of the function, and where C meets the coordinate axes.



Hint arcsin needs an input from −1 to 1, and gives an output from −π/2 to π/2.

B5 The curve C has equation $y = 3\arccos(2x) - \pi$. Find the domain and range of the function, and the coordinates of the points where C meets the coordinate axes. [6 marks]

Answer .....

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C1 Sam says $\arccos\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{6}$. Explain why Sam is wrong and give the correct value. [4 marks]

Answer .....

C2 Solve $3\arccos(2x-1) = \pi$. [4 marks]

Answer .....

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C3

(a) Solve $4 \arctan x = \pi$. [2 marks]

Answer .....

(b) Explain why $4 \arctan x = 3\pi$ has no solution. [2 marks]

Total for C3: 4 marks

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C4 Prove that $\arcsin x + \arccos x = \dfrac{\pi}{2}$ for $-1 \leq x \leq 1$. [5 marks]

C5 Find the exact values of $\arcsin(\sin 2)$ and $\arccos(\cos 4)$, where the angles are in radians. [4 marks]

Answer .....

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C6 The curve C has equation $y = 2\arctan x + \frac{\pi}{2}$. State the range of the function, the equations of the asymptotes of C, and where C meets the y-axis. [4 marks]

Answer .....

D Exam-style questions

Edexcel exam style

D1 The function f is defined by $f(x) = 2\arccos(x - 1)$, $0 \leq x \leq 2$.

(a) State the range of f. [2 marks]

Answer .....

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(b) Find $f^{-1}(x)$ and state its domain. [4 marks]

Answer .....

(c) Hence solve $f(x) = \dfrac{4\pi}{3}$. [3 marks]

Answer .....

Total for D1: 9 marks

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D2

> Ella writes: "$\cos\left(\arccos\left(-\frac{7}{25}\right)\right) = -\frac{7}{25}$ is negative, so $\sin\left(\arccos\left(-\frac{7}{25}\right)\right) = -\frac{24}{25}$."

Ella is wrong.

(a) Explain Ella's mistake and give the correct value. [2 marks]

(b) Find the exact value of $\tan\left(\arccos\left(-\frac{7}{25}\right)\right)$. [2 marks]

Answer .....

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(c) Show that $\tan(\arcsin x) = \dfrac{x}{\sqrt{1-x^2}}$ for $-1 < x < 1$. [3 marks]

(d) Hence find the exact value of $\tan\left(\arcsin\dfrac{1}{3}\right)$, with a rational denominator. [2 marks]

Answer .....

Total for D2: 9 marks

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D3 Solve $\arcsin x = \arccos 2x$, giving your answer as an exact value with a rational denominator. [7 marks]

Answer .....

D4

(a) Show that $\arctan x + \arctan \dfrac{1}{x} = \dfrac{\pi}{2}$ for $x > 0$. [4 marks]

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(b) Hence solve $\arctan x = 2\arctan\dfrac{1}{x}$ for $x > 0$. [3 marks]

Answer .....

(c) State the value of $\arctan x + \arctan\dfrac{1}{x}$ when $x < 0$, giving a reason. [2 marks]

Answer .....

Total for D4: 9 marks

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D5 The function h is defined by $h(x) = \pi - 3\arcsin\dfrac{x}{2}$, $-2 \leq x \leq 2$.

(a) Sketch the graph of $y = h(x)$. Show the coordinates of the end points of the graph and of the point where it meets the y-axis. [3 marks]

(b) State the range of h. [2 marks]

Answer .....

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(c) Find the exact x-coordinate of the point where the graph crosses the x-axis. [2 marks]

Answer .....

(d) Solve $h(x) = \arcsin \frac{x}{2}$. [2 marks]

Answer .....

Total for D5: 9 marks

D6 This question is about the equation $\arctan \frac{x}{2} = \arccos \frac{1}{x}$, where $|x| \geq 1$.

(a) Let $\theta = \arccos \frac{1}{x}$. Show that $\tan^2 \theta = x^2 - 1$. [2 marks]

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(b) By taking tan of both sides, solve the equation. Give your answer in exact form, and explain why any other value you find is not a solution. [5 marks]

Answer .....

Total for D6: 7 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Solve $\arcsin x = \arctan 2x$. Give exact answers. [8 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Let $\theta = \arcsin x = \arctan 2x$. Then $\sin\theta = x$ and $\tan\theta = 2x$.

Hint 2 So $\dfrac{\sin\theta}{\cos\theta} = 2\sin\theta$. Bring everything to one side and factorise; do not divide by $\sin\theta$.

Hint 3 θ must lie between $-\dfrac{\pi}{2}$ and $\dfrac{\pi}{2}$. Check each value of x in the original equation.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can state the domain and range of arcsin, arccos and arctan.	○ ○ ○
I can find exact values such as arccos(−√2/2) and sin(arccos(8/17)).	○ ○ ○
I can sketch a curve made from an inverse trig function, with its domain, range and axis crossings.	○ ○ ○
I can prove and use results such as arcsin x + arccos x = π/2.	○ ○ ○

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