

Linear and quadratic inequalities

74 marks



Hints online, answers free
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mathswariz.uk/worksheets/a-level-linear-and-quadratic-inequalities/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] Factorise $3x^2 + 4x - 15$.
- A2** Needed today [2 marks] The equation $2x^2 - 6x + k = 0$ has equal roots. Find k .

Answer

Answer $k = \dots\dots$

- A3** Last lesson [2 marks] Solve the simultaneous equations $y = 2x - 1$ and $xy = 3$.
- A4** From chapter 1 [2 marks] Write $\frac{12}{3 + \sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are integers.

Answer $(\dots, \dots)$ and $(\dots, \dots)$

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Find the set of values of x for which
 $7 - 2(x + 4) > 3x + 9$.

Hint Expand, collect the x terms on one side, and turn the sign round if you divide by a negative.

B1 Find the set of values of x for which

$$\frac{x-1}{3} - \frac{2x+1}{4} < 1.$$

Answer

EXAMPLE 2

Find the set of values of x for which
 $4 + 7x - 2x^2 < 0$.

Hint Solve $4 + 7x - 2x^2 = 0$ to get the critical values. The curve is an upside-down U: where is it below the x -axis?

B2 Find the set of values of x for which
 $x(11 - 3x) \geq 6$.

Answer

EXAMPLE 3

The equation $2x^2 + (k - 1)x + 2 = 0$, where k is a constant, has no real roots. Find the set of possible values of k .

Hint No real roots means $b^2 - 4ac < 0$. That gives a quadratic inequality in k .

B3 The equation [5 marks]

$(k + 1)x^2 - 4x + (k - 2) = 0$, where k is a constant, has two distinct real roots. Find the set of possible values of k .

Answer

EXAMPLE 4

Find the set of values of x for which $\frac{12}{x - 1} > 4$, where $x \neq 1$.

Hint You do not know if $x - 1$ is positive or negative, so multiply both sides by $(x - 1)^2$, which is positive.

B4 Find the set of values of [4 marks]

x for which $\frac{10}{x + 3} \leq 2$, where $x \neq -3$.

Answer

- C1** Find the set of values of x for which both $2(x + 3) > 9 - x$ and $2x^2 + 3x - 20 \leq 0$. [5 marks]

Answer

- C2** Find the set of values of x for which $(x + 4)(x - 2) > 3x + 2$. Give your critical values in surd form. [4 marks]

Answer

- C3** Find the set of values of x for which $3x - 1 < x - 7$ or $4(x - 2) > 2x + 5$. Write your answer in set notation. [3 marks]

Answer

D1 Leo writes: $(2x - 7)(x + 2) = 0$ gives critical values $\frac{7}{2}$ and -2 , so $x < -2$ or $x > \frac{7}{2}$.

Leo is finding the set of values of x for which $14 + 3x - 2x^2 > 0$.

(a) Explain the mistake Leo has made. You may use a value of x to help you. [2 marks]

(b) Write down the correct set of values of x . [1 mark]

Answer

(c) Hence find the set of values of x for which both $14 + 3x - 2x^2 > 0$ and $3(x + 2) > 7x - 2$. [3 marks]

Answer

(d) Write down the integers that satisfy both inequalities in part (c). [2 marks]

Answer

Total for D1: 8 marks

D2 The equation $2x^2 + (k - 4)x + k - 1 = 0$, where k is a constant, has two distinct real roots.

(a) Show that $k^2 - 16k + 24 > 0$.

[3 marks]

(b) Hence find the set of possible values of k . Give your answer in surd form.

[4 marks]

Answer

(c) k is a positive integer less than 20. Find all the possible values of k .

[2 marks]

Answer

Total for D2: 9 marks

D3 The fuel cost of a car journey, C pence per km, at a steady speed of v km/h is modelled by $C = 0.01v^2 - 1.2v + 50$, for $20 \leq v \leq 110$.

(a) Show that $C < 18$ when $v^2 - 120v + 3200 < 0$.

[2 marks]

(b) Hence find the range of speeds for which $C < 18$.

[3 marks]

Answer

(c) On one road the greatest speed allowed is 70 km/h. Write down the speeds on this road for which $C < 18$. [1 mark]

Answer

(d) Find the least value of C , and the speed at which it happens. [2 marks]

Answer pence per km $v = \dots\dots$

Total for D3: 8 marks

D4 In this question $x \neq 3$.

(a) Find the set of values of x for which $\frac{2x+1}{x-3} > 1$. [4 marks]

Answer

- (b) Hence find the set of values of x for which both $\frac{2x+1}{x-3} > 1$ and $x^2 - 2x - 48 \leq 0$. [3 marks]

Answer

Total for D4: 7 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The equation $x^2 - 2kx + 2k + 15 = 0$, where k is a constant, has two distinct positive roots. Find the set of possible values of k . [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Two distinct real roots means the discriminant is positive.
- Hint 2** Sketch $y = x^2 - 2kx + 2k + 15$. For both roots to be positive, the curve must cross the y -axis above O and its turning point, at $x = k$, must be to the right of the y -axis.
- Hint 3** $4k^2 - 4(2k + 15) > 0$ gives $(k - 5)(k + 3) > 0$. Now add $2k + 15 > 0$ and $k > 0$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can solve a linear inequality. ☐ ☐ ☐

I can solve a quadratic inequality with a sketch. ☐ ☐ ☐

I can solve an inequality with x in the denominator. ☐ ☐ ☐

I can find the values of k for a number of roots. ☐ ☐ ☐