Locating roots

114 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-locating-roots/

Date

A Do Now

Quick questions to start.

A1 Needed today [2 marks]

Given $\mathrm{f}(x) = x^3 - 5x + 1$, work out $\mathrm{f}(2)$ and $\mathrm{f}(2.5)$.

Answer .....

A2 Needed today [3 marks]

Find the coordinates of the turning points of $y = x^3 - 3x + 1$.

Answer .....

A3 Common slip [2 marks]

Given $\mathrm{g}(x) = 2\sin x - x$, work out $\mathrm{g}(1.8)$ and $\mathrm{g}(1.9)$ to 3 significant figures. Use radians.

Answer .....

A4 Needed today [2 marks]

Find the exact x-coordinate of the stationary point of $y = e^x - 4x$.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Show that $x^3 - 2x - 5 = 0$ has a root in the interval $[2, 3]$.

Hint Work out $f(2)$ and $f(3)$. Then write all three parts of the conclusion: sign change, continuous, root.

B1 Show that $\ln x + x - 4 = 0$ has a root in the interval $[2.9, 3]$. [3 marks]

EXAMPLE 2

The equation $x^3 - 2x - 5 = 0$ has a root α. Show that $\alpha = 2.09$ to 2 decimal places.

Hint Every number that rounds to 2.09 lies between 2.085 and 2.095. Test those two values.

B2 The equation $\ln x + x - 4 = 0$ has a root β. Show that $\beta = 2.926$ to 3 decimal places. [4 marks]

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EXAMPLE 3

Given $\mathrm{f}(x) = \dfrac{3}{x-2} + x$, show that $\mathrm{f}(1) < 0$ and $\mathrm{f}(3) > 0$. Explain why $\mathrm{f}(x) = 0$ has no root in $[1, 3]$.

Hint Where is f not defined? A change of sign only proves a root when the function is continuous.

B3 Given $\mathrm{g}(x) = \dfrac{2}{x-1} + x$, show that $\mathrm{g}(0)$ and $\mathrm{g}(2)$ have different signs. Explain why this does not prove a root in $[0, 2]$, and show there is no root. [5 marks]

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EXAMPLE 4

Given $\mathrm{f}(x) = x^3 - 3x + 2$, show that $\mathrm{f}(0.5)$ and $\mathrm{f}(1.5)$ are both positive, but that $\mathrm{f}(x) = 0$ has a root in $[0.5, 1.5]$.

Hint Factorise. A repeated root touches the axis, so the sign does not change there.

B4 Given $\mathrm{h}(x) = x^3 - 12x + 16$, show that $\mathrm{h}(1.5)$ and $\mathrm{h}(2.5)$ are both positive. Show that $\mathrm{h}(x) = 0$ has a root in $[1.5, 2.5]$ all the same. [4 marks]

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EXAMPLE 5

Show that $e^x + x^2 - 4 = 0$ has exactly one positive root, and that it lies in $[1, 1.1]$.

Hint For $x > 0$ the function is increasing. Then use a sign change on $[1, 1.1]$.

B5 Show that $\ln x + x^2 - 3 = 0$ has exactly one root, and that it lies in $[1.5, 1.6]$. [5 marks]

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C1 Ola tests $\mathrm{f}(x) = 3\sin x - x$ at $x = 2.2$ and $x = 2.3$ and gets -2.08 and -2.18. She says there is no root in $[2.2, 2.3]$. Find her mistake and show that there is a root. [4 marks]

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C2 The root α of $3\sin x - x = 0$ near 2.3 is to be given to 2 decimal places. Sam tests $x = 2.27$ and $x = 2.28$ and finds a sign change. Explain why this does not tell him α to 2 decimal places, then find it. [5 marks]

Answer .....

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C3 Given $\mathrm{f}(x) = x^3 - 6x^2 + 9x - 1$, show that $\mathrm{f}(0)$ and $\mathrm{f}(4)$ have different signs. Then show that $\mathrm{f}(x) = 0$ has three roots in $[0, 4]$ and give an interval of width 1 for each. [5 marks]

Answer .....

C4 Given $\mathrm{f}(x) = \tan x - x$, show that $\mathrm{f}(1.5) > 0$ and $\mathrm{f}(1.6) < 0$. Explain why there is no root in $[1.5, 1.6]$. [4 marks]

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C5 Given $\mathrm{f}(x) = 4x^3 - 12x^2 + 9x$, Kai works out $\mathrm{f}(1) = 1$ and $\mathrm{f}(2) = 2$ and says that $\mathrm{f}(x) = 0$ has no root in $[1, 2]$. Show that he is wrong. [4 marks]

D Exam-style questions

Edexcel exam style

D1 $\mathrm{f}(x) = \ln(2x) - x^2 + 3$, $x > 0$.

(a) Find the exact x-coordinate of the stationary point of $y = \mathrm{f}(x)$. [3 marks]

Answer .....

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(b) Show that $\mathrm{f}(x) = 0$ has exactly two roots. [3 marks]

(c) The larger root is α. Show that α lies in $[2.1, 2.2]$. [2 marks]

(d) Show that $\alpha = 2.11$ to 2 decimal places. [3 marks]

Total for D1: 11 marks

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D2

> Leo writes: "$\mathrm{f}(1.73) = -0.0923$ and $\mathrm{f}(1.74) = 0.00802$, so the root of $x^3 + x - 7 = 0$ is 1.74 to 2 decimal places."

The root is 1.74 to 2 decimal places, but Leo's working does not prove it.

(a) Explain why Leo's working is not enough. [2 marks]

(b) Show that the root is 1.74 to 2 decimal places. [3 marks]

(c) Show that this is the only root of $x^3 + x - 7 = 0$. [2 marks]

Total for D2: 7 marks

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D3 The curves $y = e^x$ and $y = 3x^2$ meet at a point where $x = \beta$ and $3.7 < \beta < 3.8$. Show that β lies in this interval and find β to 2 decimal places, showing that your answer is correct to this accuracy. [7 marks]

Answer .....

D4 $\mathrm{f}(x) = x^3 - 7x + 7$.

(a) Show that $\mathrm{f}(x) = 0$ has a root in $[-3.1, -3]$. [2 marks]

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(b) Find $\mathrm{f}(1)$ and $\mathrm{f}(2)$. Explain why these alone do not show whether there are roots between 1 and 2. [2 marks]

(c) Find the minimum value of $\mathrm{f}(x)$ for $x > 0$ and hence show that there are two roots in $[1, 2]$. [4 marks]

(d) Show that the smaller of these two roots lies in $[1.35, 1.36]$. [2 marks]

Total for D4: 10 marks

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D5 The temperature of a cup of tea, θ in °C above the room, t minutes after it is made, is modelled by $\theta = 70e^{-0.1t} - 10$. A second model for a cup that is reheated gives $\theta = 4t$. The two models give the same temperature at $t = T$.

(a) Show that T lies in $[6.5, 6.6]$. [3 marks]

(b) Show that $T = 6.57$ to 2 decimal places. [3 marks]

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(c) Find the temperature above the room at this time. [2 marks]

Answer .....

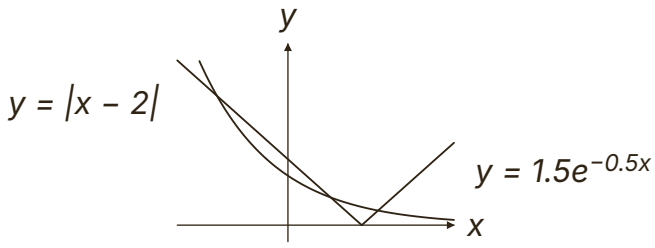
(d) Show that the two models agree at only one time. [2 marks]

Total for D5: 10 marks

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D6 The graphs of $y = 1.5e^{-0.5x}$ and $y = |x - 2|$ are shown. Let $\mathrm{f}(x) = 1.5e^{-0.5x} - |x - 2|$.

(a) Explain how the graph shows that $1.5e^{-0.5x} = |x - 2|$ has three roots, and say roughly where each one is. [2 marks]



(b) Show that one root lies in $[2.4, 2.5]$. [3 marks]

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(c) Find the negative root to 2 decimal places, showing that your answer is correct. [3 marks]

Answer .....

Total for D6: 8 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 $\mathrm{f}(x) = x^5 - 5x + k$, where k is a constant. Find the set of values of k for which $\mathrm{f}(x) = 0$ has three real roots. For $k = 1$, find an interval of width 0.1 containing each root. [9 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 $\mathrm{f}'(x) = 5x^4 - 5$, so the stationary points are at $x = \pm 1$.

Hint 2 Three roots need the maximum above the axis and the minimum below it: $\mathrm{f}(-1) > 0$ and $\mathrm{f}(1) < 0$.

Hint 3 For $k = 1$, one root is in $[-2, -1]$, one in $[0, 1]$ and one in $[1, 2]$. Narrow each with more sign changes.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can show that a root lies in an interval, stating the change of sign and that the function is continuous.	○ ○ ○
I can show that a root is correct to a given number of decimal places by testing the bounds.	○ ○ ○
I can explain why an asymptote gives a sign change with no root.	○ ○ ○
I can explain why an interval with no sign change may still contain roots.	○ ○ ○

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