



Logarithms and non-linear data

70 marks

Date _____

A Do Now

Quick questions to start.

- | | |
|---|--|
| <p>A1 [Last lesson] [2 marks]</p> <p>Solve $10^x = 250$, giving your answer to 3 significant figures.</p> | <p>A2 [This chapter] [2 marks]</p> <p>Write $\log 5 + 3 \log x$ as a single logarithm.</p> |
|---|--|

Answer

Answer

- | | |
|--|--|
| <p>A3 [GCSE Higher] [2 marks]</p> <p>A straight line passes through $(0, 1.4)$ and $(5, 2.9)$. Find its equation.</p> | <p>A4 [Needed today] [2 marks]</p> <p>Write $\ln(6e^{2x})$ in the form $p + qx$, where p and q are exact.</p> |
|--|--|

Answer

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

Which graph of logs is a straight line?
Give its gradient and vertical intercept to 3 s.f. (a) $y = 5x^3$ (b) $N = 40 \times 1.3^t$

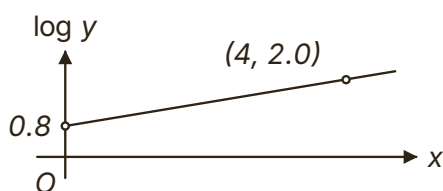
Hint Here $\log$ means $\log_{10}$. Take $\log$ of both sides and use the laws. In (b) the power law brings down t , not $\log t$.

B1 Which graph of logs is a straight line? [3 marks]
Give its gradient and vertical intercept to 3 s.f. (a)
 $T = 12x^{2.5}$ (b)
 $P = 320 \times 0.85^t$

Answer

EXAMPLE 2

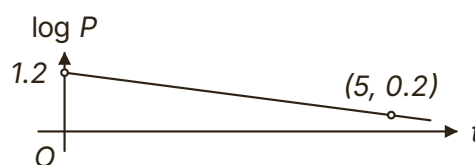
$y = ab^x$. The graph of $\log y$ against x is a straight line through $(0, 0.8)$ and $(4, 2.0)$. Find a and b to 3 s.f. What does a mean?



Not drawn accurately

Hint $\log y = \log a + x \log b$. So the gradient is $\log b$ and the intercept is $\log a$.

B2 A population is $P = ab^t$ [3 marks]
after t years. The graph of $\log P$ against t is a straight line through $(0, 1.2)$ and $(5, 0.2)$. Find a and b to 3 s.f. What does a mean?

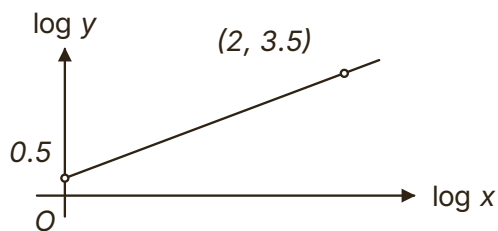


Not drawn accurately

Answer

EXAMPLE 3

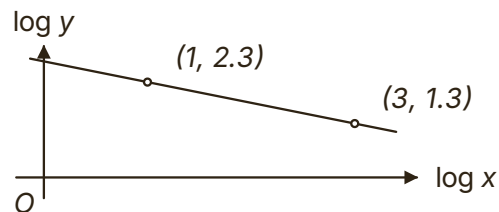
$y = ax^n$. The graph of $\log y$ against $\log x$ is a straight line through $(0, 0.5)$ and $(2, 3.5)$. Find n , and a to 3 s.f.



Not drawn accurately

Hint $\log y = \log a + n \log x$. So the gradient is n and the intercept is $\log a$.

B3 $y = ax^n$. The graph of $\log y$ against $\log x$ is a straight line through $(1, 2.3)$ and $(3, 1.3)$. Find n , and a to 3 s.f. [3 marks]



Not drawn accurately

Answer

c Practice

A-Level

C1 The data in the table follow a model of the form $y = ab^x$. By finding $\log y$ for the first and last points, find a and b to 2 significant figures. [4 marks]

x	1	3	5
y	6.0	13.5	30.4

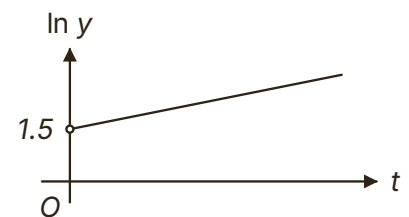
Answer

- C2** Sam has data that follow $y = 5 \times 3^x$. He says: "If I plot $\log y$ against $\log x$, the points will lie on a straight line." [4 marks]

Explain why Sam is wrong. State which graph is a straight line, and give its gradient and intercept to 3 significant figures.

Answer

- C3** A quantity is modelled by $y = Ae^{kt}$. The graph of $\ln y$ against t is a straight line with gradient 0.4 that meets the vertical axis at 1.5. Find k , and find A to 3 significant figures. Hence find y when $t = 2$. [4 marks]



Not drawn accurately

Answer

D Exam-style questions

Edexcel exam style

- D1** The value, V pounds, of a painting t years after it is bought is shown in the table. The value is modelled by $V = ab^t$, where a and b are constants.

- (a) Show that [2 marks]
 $\log V = \log a + t \log b$.

t	0	10	20	30
V	1200	2150	3850	6890

- (b) Find $\log V$, to 2 decimal places, for each value of t . Using the values of $\log V$ when $t = 0$ and $t = 30$, find a and b to 3 significant figures. Hence say what the value of b tells you about the value of the painting. [6 marks]

Answer

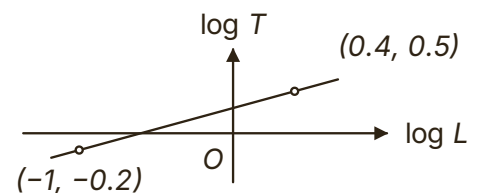
Total for D1: 8 marks

D2

A pendulum of length L m has period T s, where $T = aL^n$. The graph of $\log T$ against $\log L$ is a straight line through $(-1, -0.2)$ and $(0.4, 0.5)$. Kai says: "So $\log a = -0.2$ and $a = 0.631$."

Kai is wrong.

- (a) Explain Kai's mistake. Find the correct values of n and a , giving a to 3 significant figures. [4 marks]



Not drawn accurately

Answer

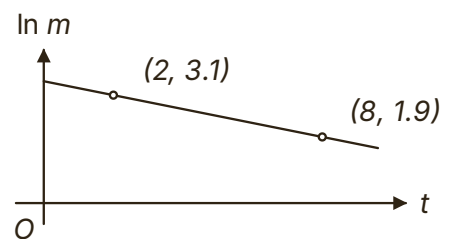
- (b) Hence find, to 3 significant figures, the length of a pendulum with a period of 3 seconds, and the period of a pendulum 0.5 m long. [4 marks]

Answer

Total for D2: 8 marks

D3 The mass, m mg, of a drug in a patient's blood t hours after it is given is modelled by $m = Ae^{-kt}$, where A and k are positive constants. The graph of $\ln m$ against t is a straight line through $(2, 3.1)$ and $(8, 1.9)$.

- (a) Show that $\ln m = \ln A - kt$. [6 marks]
Find k , and find A to 3 significant figures.



Not drawn accurately

Answer

- (b) Hence find the time when the mass first falls below 5 mg, to 3 significant figures. [3 marks]

Answer

Total for D3: 9 marks

D4 An experiment gives the results in the table. The results are thought to follow either $y = ax^n$ or $y = ab^x$.

- (a) Find $\log x$ and $\log y$ for each pair, [4 marks]
to 2 decimal places. Use them to decide which model fits, giving a reason.

x	1	2	4	8
y	2.5	4.35	7.58	13.2

Answer

- (b) Find n and a to 2 significant figures. [3 marks]

Answer

- (c) Hence estimate y when $x = 6$ and when $x = 50$, to 3 significant figures. Say which estimate is more reliable, and why. [2 marks]

Answer

Total for D4: 9 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** $y = ax^n$, and the graph of $\log y$ against $\log x$ passes through $(\log 4, \log 24)$ and $(\log 9, \log 81)$. Without a calculator, find a and n exactly, and find x when $y = 375$. [7 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Both points are on the curve: $24 = a \times 4^n$ and $81 = a \times 9^n$. Divide to remove a .
- Hint 2** $\frac{81}{24} = \left(\frac{9}{4}\right)^n$, which is $\left(\frac{3}{2}\right)^3 = \left(\frac{3}{2}\right)^{2n}$.
- Hint 3** So $n = \frac{3}{2}$ and $24 = 8a$. Then solve $3x^{3/2} = 375$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can make $y = ax^n$ a straight line: $\log y$ against $\log x$.



I can make $y = ab^x$ a straight line: $\log y$ against x .



I can find a model's constants from a graph of logs.



I can use a model and judge its predictions.

