



Logarithms and the laws of logarithms

70 marks

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] Solve $3^x = \frac{1}{27}$.
- A2** Last lesson [2 marks] $V = 400e^{-0.2t}$. Find the rate of change of V when $t = 3$.

Answer

Answer

- A3** From chapter 1 [2 marks] Find the value of $16^{\frac{3}{4}}$ and of $16^{-\frac{1}{2}}$.
- A4** GCSE Higher [2 marks] Solve $x^2 - 3x - 10 = 0$.

Answer

Answer

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

(a) Write $5^4 = 625$ as a logarithm. (b) Without a calculator, find $\log_4 \frac{1}{8}$.

Hint $\log_a n = x$ means $a^x = n$. Write 4 and $\frac{1}{8}$ as powers of 2.

B1 (a) Write $\log_6 216 = 3$ [3 marks] as a power. (b) Without a calculator, find $\log_8 \frac{1}{4}$.

Answer

EXAMPLE 2

Show that $3 \log_2 6 - \log_2 27 = 3$.

Hint Use the power law first, then the division law. Then ask: 2 to what power gives the number inside?

B2 Write $2 \log_5 10 - \log_5 4$ [3 marks] as a single logarithm, and hence find its value.

Answer

EXAMPLE 3

Write $\log_a \left(\frac{x^3 \sqrt{y}}{a^2} \right)$ in terms of $\log_a x$ and $\log_a y$.

Hint Split it with the multiplication and division laws first. $\sqrt{y} = y^{\frac{1}{2}}$, and $\log_a a = 1$.

B3 Write $\log_a \left(\frac{ax^2}{y^4} \right)$ in [3 marks] terms of $\log_a x$ and $\log_a y$.

Answer

EXAMPLE 4

Solve $\log_2(x + 10) - \log_2(x - 4) = 3$.

Hint Use the division law to make one logarithm, then rewrite it as a power. Check the answer keeps $x - 4$ positive.

B4 Solve [3 marks]

$$\log_3(5x + 8) - \log_3(x - 4) = 2.$$

Answer

c Practice

A-Level

C1 Without a calculator, explain why $\log_3 50$ lies between 3 and 4. Then use your calculator to find $\log_3 50$ to 3 decimal places. [3 marks]

Answer

C2 Solve $\log_x(3x + 10) = 2$. [3 marks]

Answer

- C3** The positive numbers a and b satisfy $\log_2 a + \log_2 b = 5$ and $a - b = 4$. Find a and b . [5 marks]

Answer $a = \dots\dots$ $b = \dots\dots$

- C4** By writing $\log_a x = m$, prove that $\log_a(x^k) = k \log_a x$. [3 marks]

D Exam-style questions

Edexcel exam style

- D1** An equation is $2 \log_4(x - 2) = 1 + \log_4 x$.
- (a) Show that the equation can be written as $x^2 - 8x + 4 = 0$. [4 marks]

- (b) Hence solve the equation, giving your answer in the form $a + b\sqrt{3}$. Explain why the equation has only one solution.

[4 marks]

Total for D1: 8 marks

D2

Sam solves $\log_2 x + \log_2(x + 2) = 3$ like this: "
 $\log_2 x + \log_2(x + 2) = \log_2(2x + 2)$, so $2x + 2 = 8$ and $x = 3$."

Sam is wrong.

- (a) Show that $x = 3$ does not satisfy the equation.

[2 marks]

- (b) Explain Sam's mistake and solve the equation correctly.

[4 marks]

Answer

Total for D2: 6 marks

D3 The positive numbers x and y satisfy the simultaneous equations

$$2 \log_4 y - \log_4 x = 1 \quad x + y = 15$$

(a) Show that $y^2 = 4x$. [3 marks]

(b) Hence find the values of x and y . [5 marks]

Answer $x = \dots\dots$ $y = \dots\dots$

Total for D3: 8 marks

D4 Given that $\log_a 2 = p$ and $\log_a 3 = q$,

(a) write $\log_a 72$ in terms of p and q . [2 marks]

Answer

(b) write $\log_a \left(\frac{27}{4a} \right)$ in terms of p and q .

[3 marks]

Answer

(c) Given also that $a = 6$, show that $p + q = 1$. Hence write $\log_6 72$ in terms of p only. [3 marks]

Answer

Total for D4: 8 marks

E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 Find the two values of x for which $\log_2 x = \log_x 16$.

[6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** Call $\log_2 x = m$, so $x = 2^m$. Now write $\log_x 16 = n$ as a power too.
- Hint 2** $x^n = 16$ gives $(2^m)^n = 2^4$, so $mn = 4$. The equation says $m = n$.
- Hint 3** $m^2 = 4$, so $m = 2$ or $m = -2$. Then $x = 2^2 = 4$ or $x = 2^{-2} = \frac{1}{4}$. Both are positive and not 1, so both are allowed as a base.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can change between a logarithm and a power, and find a logarithm without a calculator. ○ ○ ○

I can use the three laws of logarithms, including the facts that a logarithm of its own base is 1 and a logarithm of 1 is 0. ○ ○ ○

I can solve equations with logarithms and reject a value that makes a logarithm undefined. ○ ○ ○

I can write logarithms in terms of given letters, such as p and q. ○ ○ ○