

Magnitude, direction and position vectors

70 marks



Hints online, answers free
with an account:
mathswariz.uk/worksheets/a-level-magnitude-direction-and-position/

Date _____

A Do Now

Quick questions to start.

- A1** Needed today [2 marks] Simplify $\sqrt{72} + \sqrt{50}$.
- A2** Last lesson [2 marks] $\mathbf{a} = 3\mathbf{i} - 2\mathbf{j}$ and $\mathbf{b} = -\mathbf{i} + 4\mathbf{j}$. Find $2\mathbf{a} - 3\mathbf{b}$.

Answer

Answer

- A3** From chapter 9 [2 marks] Find the area of a triangle with sides 5 cm and 8 cm and 30° between them.
- A4** From chapter 10 [2 marks] Solve $\tan \theta = -1$ for $0 \leq \theta < 360^\circ$.

Answer cm^2

Answer $\theta = \dots\dots$

B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

$\mathbf{a} = 20\mathbf{i} - 21\mathbf{j}$ and $\mathbf{b} = -4\mathbf{i} + 9\mathbf{j}$. Find (a) $|\mathbf{a}|$ (b) a unit vector in the direction of $\mathbf{a}$ (c) the exact value of $|\mathbf{a} + 2\mathbf{b}|$.

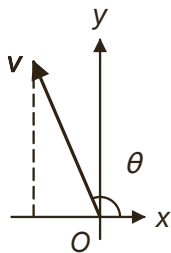
Hint $|x\mathbf{i} + y\mathbf{j}| = \sqrt{x^2 + y^2}$. A unit vector has magnitude 1: divide the vector by its own magnitude. Combine first, then find the magnitude.

B1 $\mathbf{p} = -12\mathbf{i} + 35\mathbf{j}$ and $\mathbf{q} = 6\mathbf{i} - 13\mathbf{j}$. Find (a) $|\mathbf{p}|$ (b) a unit vector in the direction of $\mathbf{p}$ (c) the exact value of $|\mathbf{p} + 3\mathbf{q}|$. [5 marks]

Answer (c)

EXAMPLE 2

For $\mathbf{v} = -3\mathbf{i} + 7\mathbf{j}$, find, to 1 d.p., the angle between $\mathbf{v}$ and (a) $\mathbf{i}$ (b) $\mathbf{j}$.



Not drawn accurately

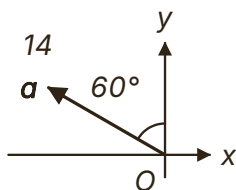
Hint The triangle has sides 3 and 7. The angle with $\mathbf{i}$ is measured from the positive x -axis: here it is over 90° .

B2 For $\mathbf{w} = 4\mathbf{i} - 9\mathbf{j}$, find, to 1 d.p., the angle between $\mathbf{w}$ and (a) $\mathbf{i}$ (b) $\mathbf{j}$. [4 marks]

Answer (a) (b)

EXAMPLE 3

a has magnitude 14 and makes 60° with the positive y -axis, as shown.
Find **a** in exact **i, j** form.



Not drawn accurately

Hint Up: $14 \cos 60^\circ$. Across: $14 \sin 60^\circ$, to the left, so the **i** part is negative.

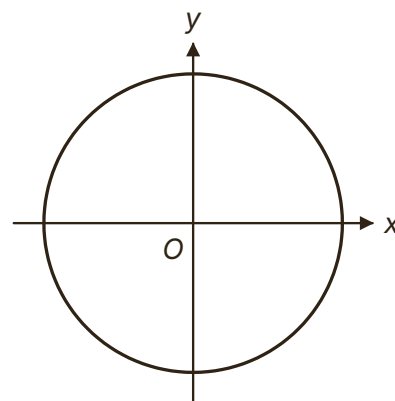
B3 **b** has magnitude 18, [3 marks]
makes 60° with the
positive x -axis and
points below the x -axis.
Find **b** in exact **i, j** form.

Answer **b** = ...

c Practice

A-Level

C1 The point A lies on the circle with equation $x^2 + y^2 = 50$. Given that [4 marks]
 $\overrightarrow{OA} = 3k\mathbf{i} - k\mathbf{j}$, find the exact possible values of k , and the possible
position vectors of A .



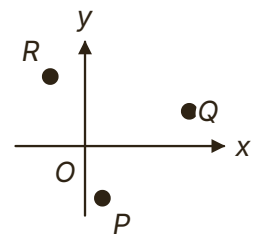
Answer k = ... $\overrightarrow{OA}$ = ...

- C2** The vector $\mathbf{a} = p\mathbf{i} + q\mathbf{j}$ has magnitude 61. The angle between $\mathbf{a}$ and $\mathbf{i}$ is θ , where $\cos \theta = \frac{11}{61}$. Find the possible values of p and q . [4 marks]

Answer $p = \dots\dots\dots$ $q = \dots\dots\dots$

- C3** The points P , Q and R have position vectors $\begin{pmatrix} 1 \\ -3 \end{pmatrix}$, $\begin{pmatrix} 6 \\ 2 \end{pmatrix}$ and $\begin{pmatrix} -2 \\ 4 \end{pmatrix}$. [4 marks]

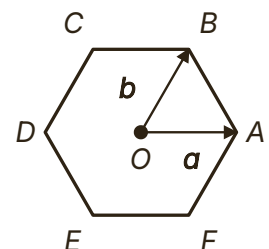
A fourth point S is chosen so that the four points are the vertices of a parallelogram. Find every possible position vector of S .



Not drawn accurately

Answer $\dots\dots\dots$ $\dots\dots\dots$ $\dots\dots\dots$

- C4** $ABCDEF$ is a regular hexagon with centre O . $\overrightarrow{OA} = \mathbf{a}$ and $\overrightarrow{OB} = \mathbf{b}$. [4 marks]
Find, in terms of $\mathbf{a}$ and $\mathbf{b}$, (i) $\overrightarrow{OC}$ (ii) $\overrightarrow{OE}$ (iii) $\overrightarrow{AC}$ (iv) $\overrightarrow{CF}$.



Answer (i) $\dots\dots\dots$ (ii) $\dots\dots\dots$ (iii) $\dots\dots\dots$ (iv) $\dots\dots\dots$

D1 The vector $\mathbf{v} = (k + 1)\mathbf{i} + (k - 7)\mathbf{j}$, where k is a constant.

(a) Show that $|\mathbf{v}|^2 = 2k^2 - 12k + 50$. [2 marks]

(b) Hence find the least possible value of $|\mathbf{v}|$, and the value of k at which it occurs. [3 marks]

Answer $|\mathbf{v}| = \dots\dots\dots$ $k = \dots\dots\dots$

Total for D1: 5 marks

D2

Ben writes: "To find a unit vector in the direction of $5\mathbf{i} - 2\mathbf{j}$, divide by $5 - 2 = 3$. The unit vector is $\frac{5}{3}\mathbf{i} - \frac{2}{3}\mathbf{j}$."

Ben is wrong.

(a) Show that Ben's vector is not a unit vector, and find the correct unit vector, with rational denominators. [3 marks]

Answer $\dots\dots\dots$

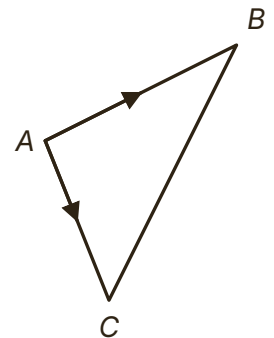
- (b) The vector $\mathbf{w}$ has magnitude $2\sqrt{58}$ and points in the opposite direction to $5\mathbf{i} - 2\mathbf{j}$. Find $\mathbf{w}$ in the form $a\sqrt{2}\mathbf{i} + b\sqrt{2}\mathbf{j}$, where a and b are integers. [3 marks]

Answer $\mathbf{w} = \dots\dots$

Total for D2: 6 marks

D3 In triangle ABC , $\overrightarrow{AB} = 6\mathbf{i} + 3\mathbf{j}$ and $\overrightarrow{AC} = 2\mathbf{i} - 5\mathbf{j}$.

- (a) Find $\overrightarrow{BC}$, and find $|\overrightarrow{BC}|$ in simplified surd form. [3 marks]



Not drawn accurately

Answer $\overrightarrow{BC} = \dots\dots$ $|\overrightarrow{BC}| = \dots\dots$

- (b) Find the size of angle BAC , in degrees, to one decimal place. [4 marks]

Answer $\dots\dots^\circ$

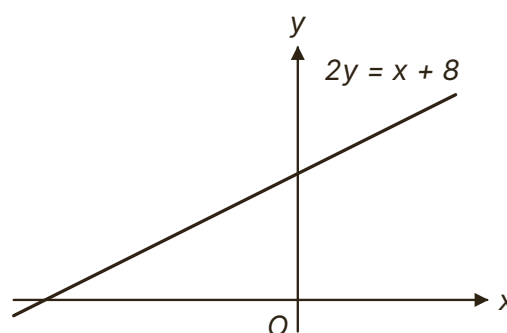
- (c) Hence find the area of triangle ABC , giving your answer to 3 significant figures. [2 marks]

Answer

Total for D3: 9 marks

D4 The point P lies on the line with equation $2y = x + 8$. The position vector of P is $\overrightarrow{OP} = x\mathbf{i} + y\mathbf{j}$, where O is the origin, and $|\overrightarrow{OP}| = 2\sqrt{13}$.

- (a) Show that [3 marks]
 $5x^2 + 16x - 144 = 0$.



- (b) Hence find the two possible position vectors of P . [3 marks]

Answer or

- (c) For the position of P with both coordinates positive, find the angle between $\overrightarrow{OP}$ and $\mathbf{j}$, to one decimal place. [2 marks]

Answer°

Total for D4: 8 marks

No route is given. The hints are at the end of the sheet. Use one at a time.

- E1** The vector $x\mathbf{i} + y\mathbf{j}$ has integer x, y and magnitude 25. How many such vectors are there? How many make an angle under 45° with $\mathbf{i}$? [6 marks]

Answer

Extension hints

Cover these with paper. Uncover one at a time.

- E1** **Hint 1** You need every integer pair with $x^2 + y^2 = 625$. Try $x = 0, 1, \dots, 25$.
- Hint 2** $625 = 0^2 + 25^2 = 7^2 + 24^2 = 15^2 + 20^2$. Now allow minus signs and swap x and y .
- Hint 3** $4 + 8 + 8 = 20$ vectors. Under 45° with $\mathbf{i}$ needs $x > 0$ and $|y| < x$.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find the magnitude of a vector and a unit vector in its direction. ○ ○ ○

I can find a vector's direction, and a vector from its magnitude and direction. ○ ○ ○

I can use position vectors to find lengths and points. ○ ○ ○

I can find a missing component from a magnitude. ○ ○ ○