Force diagrams and forces as vectors

120 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-force-diagrams-forces-as-vectors/

Date

A Do Now

Quick questions to start. Some use earlier lessons; some set a slip that costs marks.

A1 Needed today [3 marks]

The unit vectors $\mathbf{i}$ and $\mathbf{j}$ point due east and due north. Find the magnitude and the bearing of the vector $5\mathbf{i} - 12\mathbf{j}$.

Answer .....

A2 Needed today [2 marks]

A parcel has mass 750 g. Find the weight of the parcel.

Answer .....

A3 Common slip [2 marks]

A force has magnitude 20 N and acts in the direction of the vector $3\mathbf{i} - 4\mathbf{j}$. Write the force in the form $(p\mathbf{i} + q\mathbf{j})$ N.

Answer .....

A4 Common slip [2 marks]

Find the angle between the force $(-6\mathbf{i} - 8\mathbf{j})$ N and the vector $\mathbf{i}$.

Answer .....

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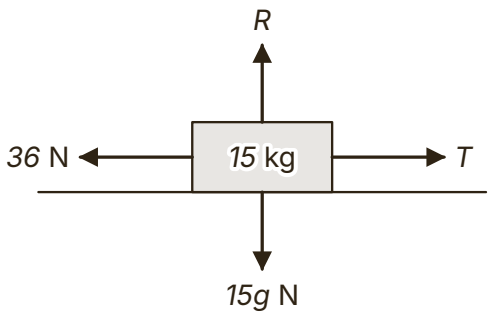
B Example, then your turn

A-Level

Your teacher works through each example with you. Then try the one beside it.

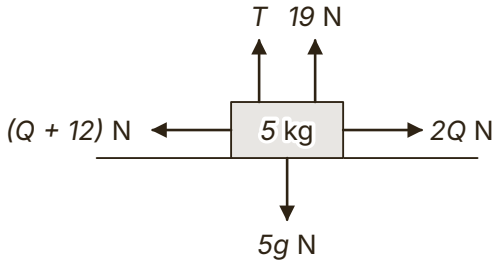
EXAMPLE 1

A sledge of mass 15 kg is pulled across horizontal snow at constant velocity by a horizontal rope. The resistance to motion is 36 N. The sledge is modelled as a particle and the rope as light. Draw a force diagram, and find the tension in the rope and the normal reaction from the snow. The tension is then increased to 50 N. Describe the motion of the sledge.



Hint Four forces: weight down, normal reaction up, tension forwards, resistance backwards. Constant velocity means the resultant force is zero, so each direction balances.

B1 A box of mass 5 kg rests on a horizontal floor. A light rope pulls vertically up on the box with tension T N, and the normal reaction from the floor is 19 N. Two horizontal forces of $2Q$ N and $(Q + 12)$ N act in opposite directions, as shown. The box stays at rest. Find T and Q. [4 marks]



Answer .....

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EXAMPLE 2

In this question $\mathbf{i}$ points due east and $\mathbf{j}$ due north. The forces $(3\mathbf{i} + 5\mathbf{j})$ N, $(2\mathbf{i} - 9\mathbf{j})$ N and $(-\mathbf{i} + \mathbf{j})$ N act on a particle. Find the resultant force, its magnitude and its bearing.

Hint Add the $\mathbf{i}$ parts and the $\mathbf{j}$ parts. Sketch the resultant before using $\tan$: a bearing is measured clockwise from north.

B2 In this question $\mathbf{i}$ points due east and $\mathbf{j}$ due north. The forces $(2\mathbf{i} + 9\mathbf{j})$ N, $(-4\mathbf{i} + 5\mathbf{j})$ N and $(-3\mathbf{i} - 2\mathbf{j})$ N act on a particle. Find the magnitude of the resultant force and its bearing. [5 marks]

Answer .....

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EXAMPLE 3

The forces $\mathbf{F}_1 = (4\mathbf{i} - 3\mathbf{j})$ N, $\mathbf{F}_2 = (-7\mathbf{i} + 2\mathbf{j})$ N and $\mathbf{F}_3 = (a\mathbf{i} + b\mathbf{j})$ N act on a particle in equilibrium. Find a and b. The force $\mathbf{F}_2$ is then removed. Find the magnitude of the resultant of the two forces that remain.

Hint In equilibrium the resultant is $0\mathbf{i} + 0\mathbf{j}$, so the third force is minus the sum of the other two. When a force is removed, the others add up to minus that force.

B3 The forces $\mathbf{F}_1 = (p\mathbf{i} + 3\mathbf{j})$ N, $\mathbf{F}_2 = (-6\mathbf{i} + q\mathbf{j})$ N and $\mathbf{F}_3 = (2\mathbf{i} - 8\mathbf{j})$ N act on a particle in equilibrium. Find p and q. The force $\mathbf{F}_1$ is then removed. Find the magnitude of the resultant of $\mathbf{F}_2$ and $\mathbf{F}_3$, and the angle it makes with $\mathbf{i}$. [5 marks]

Answer .....

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EXAMPLE 4

A force $\mathbf{F}$ has magnitude 39 N and acts in the direction of $5\mathbf{i} - 12\mathbf{j}$. Write $\mathbf{F}$ in $\mathbf{i}$, $\mathbf{j}$ form. A second force $\mathbf{G} = (p\mathbf{i} + 6\mathbf{j})$ N has magnitude 10 N, where $p < 0$. Find p, and the resultant of $\mathbf{F}$ and $\mathbf{G}$.

Hint A force of magnitude F in the direction of $\mathbf{v}$ is $\dfrac{F}{|\mathbf{v}|}\mathbf{v}$. A magnitude gives a quadratic in p: use the condition to choose the root.

B4 A force $\mathbf{F}$ has magnitude 15 N and acts in the direction of $3\mathbf{i} + 4\mathbf{j}$. A second force is $\mathbf{G} = (-4\mathbf{i} + q\mathbf{j})$ N, where $q < 0$. The resultant of $\mathbf{F}$ and $\mathbf{G}$ has magnitude 13 N. Find $\mathbf{F}$ in $\mathbf{i}$, $\mathbf{j}$ form and find q. [5 marks]

Answer .....

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EXAMPLE 5

The forces $\mathbf{F}_1 = (4\mathbf{i} - 7\mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + 2\mathbf{j})$ N act on a particle. Their resultant is parallel to the vector $3\mathbf{i} - \mathbf{j}$. Find p and the magnitude of the resultant.

Hint Parallel to $3\mathbf{i} - \mathbf{j}$ means the resultant is $k(3\mathbf{i} - \mathbf{j})$ for some number k. Find k from the component you know.

B5 [5 marks]

The forces $\mathbf{F}_1 = (3\mathbf{i} - \mathbf{j})$ N and $\mathbf{F}_2 = (p\mathbf{i} + q\mathbf{j})$ N act on a particle. Their resultant is parallel to the vector $2\mathbf{i} + 5\mathbf{j}$. Show that $5p - 2q + 17 = 0$. Given that $q = 6$, find p and the magnitude of the resultant.

Answer .....

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C1 A particle is in equilibrium under three forces, $\mathbf{F}_1 = (6\mathbf{i} - 2\mathbf{j})$ N, $\mathbf{F}_2 = (-\mathbf{i} - 10\mathbf{j})$ N and $\mathbf{F}_3$. Amir writes $\mathbf{F}_3 = \mathbf{F}_1 + \mathbf{F}_2 = (5\mathbf{i} - 12\mathbf{j})$ N. Explain his mistake, then find the magnitude of $\mathbf{F}_3$ and the angle $\mathbf{F}_3$ makes with $\mathbf{i}$. [5 marks]

Answer .....

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C2 A particle moves in a straight line at constant velocity under three forces: $(3\mathbf{i} - 4\mathbf{j})$ N, $(p\mathbf{i} + 2\mathbf{j})$ N and $(-5\mathbf{i} + q\mathbf{j})$ N. Sam says: "It is moving, so there must be a resultant force in the direction it moves." Explain why Sam is wrong, and find p and q. [4 marks]

Answer .....

C3 A lamp of mass 2.5 kg hangs at rest from two light vertical chains. The tension in one chain is 1.5 times the tension in the other. Lia writes $T + 1.5T = 2.5$, so $T = 1$ N. Explain her mistake and find the tension in each chain. [4 marks]

Answer .....

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C4 In this question $\mathbf{i}$ points due east and $\mathbf{j}$ due north. The resultant of the forces $(2\mathbf{i} + 3\mathbf{j})$ N and $(p\mathbf{i} - 7\mathbf{j})$ N has magnitude 5 N. Find the two possible values of p. Given also that the resultant acts on a bearing between $180°$ and $270°$, find p and the bearing of the resultant. [5 marks]

Answer .....

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C5 A crate of mass 40 kg is lowered by a vertical rope at a constant speed of $0.5\ \text{m s}^{-1}$. Air resistance of 6 N acts on the crate. Ben says: "The crate is moving down, so the resultant force on it is downwards." Explain why Ben is wrong, find the tension in the rope, and state one assumption made by modelling the rope as light. [5 marks]

Answer .....

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D Exam-style questions

Edexcel exam style

D1 In this question $\mathbf{i}$ points due east and $\mathbf{j}$ due north. A particle is in equilibrium under three forces, $\mathbf{F}_1 = a(2\mathbf{i} - \mathbf{j})$ N, $\mathbf{F}_2 = b(\mathbf{i} + 3\mathbf{j})$ N and $\mathbf{F}_3 = (-8\mathbf{i} - 3\mathbf{j})$ N, where a and b are constants.

(a) Find the value of a and the value of b. [3 marks]

Answer .....

(b) The force $\mathbf{F}_3$ is replaced by a force $\mathbf{G}$ of magnitude 5 N acting in the direction of $-3\mathbf{i} - 4\mathbf{j}$. Find the resultant of $\mathbf{F}_1$, $\mathbf{F}_2$ and $\mathbf{G}$, and its magnitude. [3 marks]

Answer .....

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(c) Find the bearing of this resultant. [2 marks]

Answer .....

(d) A fourth force $\mathbf{H}$ is now added, so that the resultant of $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{G}$ and $\mathbf{H}$ acts due north and has magnitude 4 N. Find $\mathbf{H}$. [2 marks]

Answer .....

Total for D1: 10 marks

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D2

The forces $\mathbf{P} = (2\mathbf{i} + 3\mathbf{j})$ N and $\mathbf{Q} = (p\mathbf{i} - 9\mathbf{j})$ N act on a particle. Their resultant is parallel to $\mathbf{i} - 2\mathbf{j}$. Kai writes: "$2 + p = 1$ and $3 - 9 = -2$, so $p = -1$."

Kai is wrong.

(a) Explain the mistake in Kai's working. [2 marks]

Answer .....

(b) Find the value of p. [3 marks]

Answer .....

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(c) Find the magnitude of the resultant. [2 marks]

Answer .....

(d) A third force $\mathbf{S}$ is added so that the particle is in equilibrium. Given that $\mathbf{i}$ points due east and $\mathbf{j}$ due north, find $\mathbf{S}$ and its bearing. [3 marks]

Answer .....

Total for D2: 10 marks

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D3 Two forces act on a particle: $\mathbf{F}_1 = (3\mathbf{i} + 4\mathbf{j})$ N and $\mathbf{F}_2$. The force $\mathbf{F}_2$ has magnitude 5 N. The resultant of $\mathbf{F}_1$ and $\mathbf{F}_2$ is parallel to $\mathbf{i}$ and has magnitude 6 N. Find $\mathbf{F}_2$ in $\mathbf{i}$, $\mathbf{j}$ form. [6 marks]

Answer .....

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D4 A crate of mass 30 kg is pushed across a horizontal floor at constant speed by a horizontal force of P N. A light rope fixed to the top of the crate pulls vertically upwards with tension 80 N. The resistance to motion is modelled as a constant force of 50 N. The crate is modelled as a particle.

(a) Draw a diagram showing all the forces acting on the crate. [2 marks]

(b) Find the normal reaction between the crate and the floor. [2 marks]

Answer .....

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(c) Find P. [2 marks]

Answer .....

(d) The tension in the rope is increased to 300 N. Explain what happens to the crate. [2 marks]

(e) State one limitation of modelling the crate as a particle. [2 marks]

Total for D4: 10 marks

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D5 In this question $\mathbf{i}$ points due east and $\mathbf{j}$ due north. The forces $\mathbf{F}_1 = (2\mathbf{i} + 7\mathbf{j})$ N, $\mathbf{F}_2$ and $\mathbf{F}_3$ act on a particle. The force $\mathbf{F}_2$ has magnitude 10 N and acts in the direction of $-3\mathbf{i} + 4\mathbf{j}$.

(a) Write $\mathbf{F}_2$ in $\mathbf{i}$, $\mathbf{j}$ form. [2 marks]

Answer .....

(b) Given that the particle is in equilibrium, find the magnitude of $\mathbf{F}_3$. [3 marks]

Answer .....

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(c) The force $\mathbf{F}_1$ is now reversed in direction, so that $-\mathbf{F}_1$ acts in place of $\mathbf{F}_1$. Show that the new resultant is $-2\mathbf{F}_1$ and find its magnitude. [2 marks]

Answer .....

(d) Find the angle between this new resultant and $\mathbf{i}$. [2 marks]

Answer .....

Total for D5: 9 marks

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D6 In this question $\mathbf{i}$ points due east and $\mathbf{j}$ due north. A ship is towed by two tugs. The first tug pulls with a force of magnitude 2600 N in the direction of $12\mathbf{i} + 5\mathbf{j}$. The second tug pulls with a force of magnitude 1500 N in the direction of $3\mathbf{i} - 4\mathbf{j}$. The ship moves in a straight line at constant velocity. The resistance of the water acts directly against the motion.

(a) Find the resultant of the two tug forces in $\mathbf{i}$, $\mathbf{j}$ form. [3 marks]

Answer .....

(b) Find the magnitude of the resistance of the water. [2 marks]

Answer .....

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(c) Find the bearing on which the ship moves. [2 marks]

Answer .....

(d) State one limitation of modelling the resistance of the water as a constant force. [2 marks]

Total for D6: 9 marks

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No route is given. The hints are at the end of the sheet. Use one at a time.

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E1 In this question $\mathbf{i}$ points due east and $\mathbf{j}$ due north. A particle is acted on by the forces $(2\mathbf{i} + 3\mathbf{j})$ N, $(-4\mathbf{i} + \mathbf{j})$ N and $\lambda(3\mathbf{i} + 4\mathbf{j})$ N, where λ is a constant. Show that the resultant force can never be zero. Find the least possible magnitude of the resultant, the value of λ that gives it, and the bearing of the resultant then. Find also the value of λ for which the resultant acts in the direction of $-\mathbf{i} - \mathbf{j}$. [10 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write the resultant as $(3\lambda - 2)\mathbf{i} + (4\lambda + 4)\mathbf{j}$ and the square of its magnitude as a quadratic in λ.

Hint 2 Complete the square: the least value of the quadratic is the square of the least magnitude. If that least value is not zero, the resultant is never zero.

Hint 3 For the direction $-\mathbf{i} - \mathbf{j}$ the two components must be equal and both negative.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can draw a force diagram and find unknown forces when a body is at rest or moves at constant velocity.	○ ○ ○
I can find the resultant of forces in i, j form and give its magnitude, its bearing and its angle with i.	○ ○ ○
I can find unknown forces for equilibrium, and the resultant when one force is removed.	○ ○ ○
I can write a force of given magnitude along a given vector, and use a resultant that is parallel to a vector.	○ ○ ○

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