Functions of time and using differentiation

123 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-functions-of-time-and-differentiation/

Date

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A Do Now

Quick questions to start. Some check what you need today; some set a slip that costs marks.

A1 Needed today [2 marks]

Find $\dfrac{\mathrm{d}y}{\mathrm{d}x}$ and $\dfrac{\mathrm{d}^2y}{\mathrm{d}x^2}$ for $y = 4x^3 - 5x^2 + 7x - 2$.

Answer .....

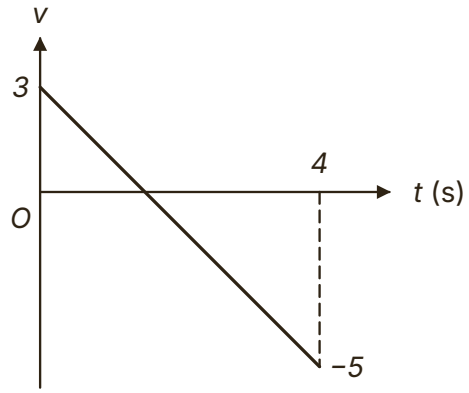
A2 Needed today [2 marks]

Solve $2t^2 - 7t - 15 = 0$, given that $t \geq 0$.

Answer .....

A3 Needed today [2 marks]

The velocity–time graph of a particle is a straight line from $3\ \mathrm{m\ s^{-1}}$ at $t = 0$ to $-5\ \mathrm{m\ s^{-1}}$ at $t = 4$. Find its acceleration and the time at which it is at rest.



Answer .....

A4 Common slip [2 marks]

A particle moving on a straight line has velocity $-6\ \mathrm{m\ s^{-1}}$. Ana says its speed is $-6\ \mathrm{m\ s^{-1}}$. Correct her, and describe the motion.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A particle moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = 2t^3 - 9t^2 + 12t$, $t \geq 0$. Find its initial velocity, the times when it is at rest, and its acceleration at each of those times.

Hint $v = \dfrac{\mathrm{d}s}{\mathrm{d}t}$ and $a = \dfrac{\mathrm{d}v}{\mathrm{d}t}$. "Initial" means $t = 0$. "At rest" means $v = 0$; it does not mean $a = 0$.

B1 A particle moves in a straight line. Its displacement from O at time t seconds is s metres, where $s = t^3 - 9t^2 + 24t + 5$, $t \geq 0$. Find the initial velocity, the times when the particle is at rest, the acceleration at each of those times, and the distance between the two points where it is at rest. [6 marks]

Answer .....

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EXAMPLE 2

A toy car leaves a point A at time $t = 0$ and moves along a straight track, then returns to A. Its distance from A at time t seconds is s metres, where $s = 6t^2 - t^3$, $0 \leq t \leq T$. Find s when $t = 2$. Find T, explaining why the model is only valid for $0 \leq t \leq T$. Find the velocity when $t = 5$ and state the direction of motion then.

Hint The car is back at A when $s = 0$. A distance from A cannot be negative, and the motion starts at $t = 0$.

B2 A model train leaves a point A at time $t = 0$ and moves along a straight track, then returns to A. Its distance from A at time t seconds is s metres, where $s = 2t^2 - 0.2t^3$, $0 \leq t \leq T$. Find T, explaining your answer. Find the velocity of the train when $t = 8$, and state its direction of motion then. [5 marks]

Answer .....

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EXAMPLE 3

A particle moves in a straight line with velocity $v\ \mathrm{m\ s^{-1}}$ at time t seconds, where $v = 2t^2 - 14t + 20$, $t \geq 0$. Find its direction of motion when $t = 3$, and find all the times at which its speed is $4\ \mathrm{m\ s^{-1}}$.

Hint Speed is the size of the velocity. A speed of 4 means $v = 4$ OR $v = -4$: solve both.

B3 A particle moves in a straight line with velocity $v\ \mathrm{m\ s^{-1}}$ at time t seconds, where $v = t^2 - 8t + 12$, $t \geq 0$. Find all the times at which its speed is $3\ \mathrm{m\ s^{-1}}$. Give exact answers. [5 marks]

Answer .....

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EXAMPLE 4

A particle moves on the x-axis. At time t seconds its displacement from O is x metres, where $x = t^3 + pt^2 + qt$ and p and q are constants. Its initial velocity is $9\ \text{m s}^{-1}$ and it is at rest when $t = 3$. Find p and q. Find the other time at which it is at rest, and its acceleration then.

Hint Differentiate first, keeping p and q. Each condition gives an equation.

B4 [6 marks]

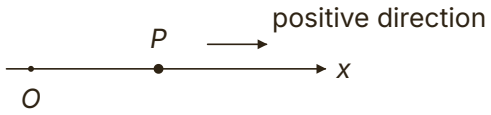
A particle moves on the x-axis. At time t seconds its displacement from O is x metres, where $x = 2t^3 + pt^2 + qt$ and p and q are constants. Its initial velocity is $24\ \text{m s}^{-1}$ and its acceleration is zero when $t = 2$. Find p and q, and show that the particle never moves in the negative x-direction.

Answer .....

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EXAMPLE 5

A particle P moves on the x-axis. At time t seconds its displacement from O is x metres, where $x = t^3 - 12t^2 + 36t$, $0 \leq t \leq 8$. Find the total distance travelled by P in the 8 seconds, and its displacement from O at $t = 8$. Find also the distance travelled in the third second.



Hint Find where P turns round ($v = 0$). Find x at $t = 0$, at each turning time and at $t = 8$, and add the sizes of the changes. The third second is from $t = 2$ to $t = 3$.

B5 A particle P moves on the x-axis. At time t seconds its displacement from O is x metres, where $x = t^3 - 9t^2 + 15t$, $0 \leq t \leq 6$. Find the total distance travelled by P in the 6 seconds, and its displacement from O when $t = 6$. [6 marks]

Answer .....

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C1 A particle moves in a straight line through a fixed point O. Its displacement from O at time t seconds is s metres, where $s = t^3 - 11t^2 + 24t$, $t \geq 0$. Sam says: "$s(7) = -28$, so when $t = 7$ the particle is moving in the negative direction." Explain why Sam is wrong. Find the interval of time during which the particle is on the negative side of O and moving towards O. [5 marks]

Answer .....

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C2 A particle moves in a straight line. Its displacement at time t seconds is s metres, where $s = 2t^3 - 15t^2 + 24t$, $t \geq 0$. Priya says: "The particle is at rest when $t = 1$, so its acceleration is zero when $t = 1$." Explain why Priya is wrong. Find the acceleration at each time the particle is at rest, and the velocity at the time when the acceleration is zero. [5 marks]

Answer .....

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C3 A particle moves in a straight line. Its displacement at time t seconds is s metres, where $s = t^3 + 4t$. Ben finds its velocity at $t = 2$ like this: "$s(2) = 16$ and the initial velocity is 4, so using $s = \frac{1}{2}(u + v)t$, $16 = \frac{1}{2}(4 + v) \times 2$, so $v = 12$." Explain why Ben's method is not valid, and find the correct velocity at $t = 2$. [4 marks]

Answer .....

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C4 A particle moves in a straight line with velocity $v \text{ m s}^{-1}$ at time t seconds, where $v = t^2 - 6t + 8$, $0 \leq t \leq 6$. Find the intervals of time in which the particle is slowing down. [6 marks]

Answer .....

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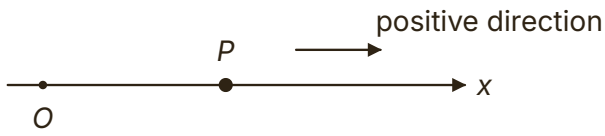
C5 A particle moves in a straight line. Its displacement at time t seconds is s metres, where $s = 2t^3 - 13.5t^2 + 21t$, $t \geq 0$. Lena says: "$s(4) - s(3) = 0.5$, so the particle travels 0.5 m in the fourth second." Find the distance the particle actually travels in the fourth second. [6 marks]

Answer .....

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D1 A particle P moves on the x-axis. At time t seconds the displacement of P from O is x metres, where $x = t^3 - 10.5t^2 + 30t$, $t \geq 0$.

(a) Find the values of t at which P is instantaneously at rest. [3 marks]



Answer .....

(b) Show that P never returns to O. [3 marks]

Answer .....

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(c) Find the speed of P at the instant when its acceleration is zero. [3 marks]

Answer .....

Total for D1: 9 marks

D2

> Kai finds when P is at rest like this: "$a = \dfrac{\mathrm{d}v}{\mathrm{d}t} = 4 - 2t = 0$, so P is at rest when $t = 2$."

A particle P moves in a straight line. At time t seconds its velocity is v m s^{-1}, where $v = 12 + 4t - t^2$, $t \geq 0$.

(a) Explain the mistake in Kai's working. [2 marks]

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(b) Find the time at which P is instantaneously at rest. [2 marks]

Answer .....

(c) Find the magnitude of the acceleration of P when it is at rest. [2 marks]

Answer .....

(d) Describe the motion of P when $t = 7$. [2 marks]

Answer .....

Total for D2: 8 marks

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D3 A particle moves in a straight line. At time t seconds its displacement from a fixed point O on the line is s metres, where $s = 2t^3 - 21t^2 + kt$, $t \geq 0$, and k is a constant. The particle is instantaneously at rest when $t = 2$. Find the total distance travelled by the particle in the first 6 seconds of its motion. [9 marks]

Answer .....

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D4 A drone takes off vertically from the ground at time $t = 0$. Its height above the ground at time t seconds is modelled as h metres, where $h = 3t^2 - 0.1t^3$, $0 \leq t \leq T$. The drone lands at time $t = T$.

(a) Find T and explain why the model is only valid for $0 \leq t \leq T$. [2 marks]

Answer .....

(b) Find the time at which the drone is instantaneously at rest, and its height then. [3 marks]

Answer .....

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(c) Find the speed of the drone when it lands. [2 marks]

Answer .....

(d) Using your answer to (c), state one criticism of the model and suggest how it could be refined. [2 marks]

Total for D4: 9 marks

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D5 A particle P moves on the x-axis. At time t seconds its displacement from O is x metres, where $x = pt^3 + qt^2$ and p and q are constants. When $t = 2$, P is instantaneously at rest at the point where $x = -8$.

(a) Find the values of p and q. [5 marks]

Answer .....

(b) Find the time at which P passes back through O. [2 marks]

Answer .....

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(c) Find the speed of P as it passes back through O. [2 marks]

Answer .....

Total for D5: 9 marks

D6 A particle P moves in a straight line. At time t seconds its velocity is $v \text{ m s}^{-1}$, where $v = (t-1)(t-3)(t-6)$, $0 \leq t \leq 7$.

(a) Sketch a velocity–time graph for $0 \leq t \leq 7$, showing the values where it meets the axes and the value of v at $t = 7$. [3 marks]

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(b) State the intervals of time in which P moves in the negative direction. [2 marks]

Answer .....

(c) Find the acceleration of P when $t = 2$. [2 marks]

Answer .....

Total for D6: 7 marks

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No route is given. The hints are at the end of the sheet. Use one at a time.

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E1 A particle P moves on the x-axis. At time t seconds, $t \geq 0$, its displacement from O is x metres, where $x = t^3 - 6t^2 + kt$ and k is a constant. Find the set of values of k for which P never moves in the negative x-direction. In the case $k = 9$, find the total distance P travels in the first 4 seconds. Find the set of values of k for which P returns to O at some time $t > 0$. [10 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Write v in completed-square form. "Never moves in the negative direction" means $v \geq 0$ for every $t \geq 0$.

Hint 2 For $k = 9$, find when $v = 0$, then the position at $t = 0$, 1, 3 and 4, and add the sizes of the moves.

Hint 3 $x = t(t^2 - 6t + k)$. You need $t^2 - 6t + k = 0$ to have a root with $t > 0$: use the discriminant and look at the larger root.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can find velocity and acceleration by differentiating, and use them at a given time.	○ ○ ○
I can find when a particle is at rest, at its start, or moving with a given speed.	○ ○ ○
I can find unknown constants in a displacement from given conditions.	○ ○ ○
I can find the total distance travelled when a particle changes direction.	○ ○ ○

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