Using integration and deriving the formulae

135 marks

Hints online, answers free with an account: mathswariz.uk/worksheets/a-level-mechanics-integration-and-deriving-the-formulae/

Date

A Do Now

Quick questions to start. Some check what you need today; some set a slip that costs marks.

A1 Needed today [2 marks]

Find $\int (6t^2 - 4t + 5)\,\mathrm{d}t$.

Answer .....

A2 Needed today [2 marks]

Given that $f'(t) = 4t - 3$ and $f(2) = 7$, find $f(t)$.

Answer .....

A3 Last lesson [2 marks]

A particle has velocity $v = t^2 - 6t + 8\ \mathrm{m\,s^{-1}}$ for $0 \leq t \leq 5$. Find its greatest speed.

Answer .....

A4 Common slip [3 marks]

A particle has velocity $v = t - 3\ \mathrm{m\,s^{-1}}$. Mo says the distance travelled from $t = 0$ to $t = 4$ is $\int_0^4 (t - 3)\,\mathrm{d}t = -4$, so $4\ \mathrm{m}$. Find the distance travelled.

Answer .....

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B Example, then your turn

Your teacher works through each example with you. Then try the one beside it.

EXAMPLE 1

A particle moves on the x-axis with acceleration $a = (6t - 4)\ \text{m s}^{-2}$ at time t seconds. When $t = 0$ it has velocity $3\ \text{m s}^{-1}$ and is at the point where $x = 2$. Find x when $t = 2$.

Hint Integrate a to get v and use $v = 3$ when $t = 0$. Then integrate v to get x and use $x = 2$ when $t = 0$. Each integral needs its own constant.

B1 [6 marks]

A particle moves on the x-axis with acceleration $a = (12 - 6t)\ \text{m s}^{-2}$ at time t seconds. When $t = 0$ it has velocity $-9\ \text{m s}^{-1}$ and is at the point where $x = 5$. Find the times at which it is at rest, its position at each of these times, and the distance between these positions.

Answer .....

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EXAMPLE 2

A particle moves in a straight line with acceleration $a = (6t + 4)\ \text{m s}^{-2}$ at time t seconds. Its displacement from O is $6\ \text{m}$ when $t = 1$ and $20\ \text{m}$ when $t = 2$. Find its displacement and velocity when $t = 3$.

Hint You are not told the initial velocity, so keep both constants: $v = 3t^2 + 4t + c$ and $s = t^3 + 2t^2 + ct + d$. The two positions give two equations in c and d.

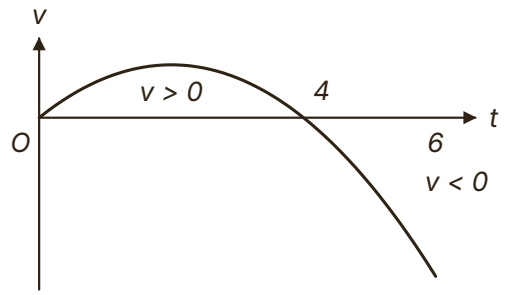
B2 A particle moves in a straight line with acceleration $a = (12t - 2)\ \text{m s}^{-2}$ at time t seconds. Its displacement from O is $4\ \text{m}$ when $t = 1$ and $17\ \text{m}$ when $t = 2$. Find its displacement and velocity when $t = 3$. [5 marks]

Answer .....

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EXAMPLE 3

A particle moves in a straight line with velocity $v = (8t - 2t^2)\ \mathrm{m\ s^{-1}}$ at time t seconds. Find the distance it travels from $t = 0$ to $t = 6$, and its displacement over this time.



Hint Find where $v = 0$ in the interval. Integrate separately on each side of that time and add the sizes.

B3 A particle moves in a straight line with velocity $v = (t^2 - 7t + 10)\ \mathrm{m\ s^{-1}}$ at time t seconds. Find the distance it travels between $t = 0$ and $t = 6$. [6 marks]

Answer .....

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EXAMPLE 4

A particle starts from rest at O and moves in a straight line. Its velocity at time t seconds is $v = 3t^2$ for $0 \leq t \leq 2$ and $v = 16 - 2t$ for $2 \leq t \leq 8$. Find the total distance it travels.

Hint Check that the two expressions give the same velocity at $t = 2$. Integrate each over its own interval. Check whether v changes sign.

B4 A particle starts from rest at O and moves in a straight line. Its velocity at time t seconds is $v = 6t - t^2$ for $0 \leq t \leq 4$ and $v = 12 - t$ for $4 \leq t \leq 12$. Find the total distance it travels, and the acceleration of the particle just before and just after $t = 4$. [6 marks]

Answer .....

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EXAMPLE 5

A particle moves in a straight line with constant acceleration a. At time $t = 0$ it is at O with velocity u. Show, by integration, that $v = u + at$ and $s = ut + \frac{1}{2}at^2$.

Hint $\frac{\mathrm{d}v}{\mathrm{d}t} = a$ with a a constant. Use the conditions at $t = 0$ to find each constant.

B5 A particle moves in a straight line with constant acceleration $-4\ \mathrm{m\ s^{-2}}$. At time $t = 0$ it is at O with velocity $10\ \mathrm{m\ s^{-1}}$. Show by integration that $v = 10 - 4t$ and $s = 10t - 2t^2$, and hence show that $v^2 = 100 - 8s$. [6 marks]

Answer .....

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C1 A particle starts from rest at O and moves in a straight line with acceleration $a = 2t \text{ m s}^{-2}$. Zoe finds its displacement at $t = 4$ like this: "$a = 8$ when $t = 4$, so $s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}(8)(16) = 64 \text{ m}$." Explain why Zoe's method is wrong, and find the correct displacement. [5 marks]

Answer .....

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C2 A particle moves on the x-axis with velocity $v = (3t^2 - 2)\ \mathrm{m\ s^{-1}}$. When $t = 1$ it is at the point where $x = 5$. Tia writes $x = t^3 - 2t$ and gives $x(3) = 21$. Find the correct value of x when $t = 3$. [5 marks]

Answer .....

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C3 A particle moves in a straight line with velocity $v = (3t^2 - 15t + 12)\ \mathrm{m\ s^{-1}}$. Find $\displaystyle\int_0^5 v\,\mathrm{d}t$ and the distance travelled between $t = 0$ and $t = 5$, and explain why they are different. [6 marks]

Answer .....

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C4 Two particles move in straight lines. The displacement of P is $s = 5 + 12t - 2t^2$ metres and the displacement of Q is $s = 12t - 2t^3$ metres, at time t seconds. Show that P moves with constant acceleration, stating its initial velocity and its acceleration. Explain why the constant acceleration formulae cannot be used for Q. [5 marks]

Answer .....

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C5 A particle moves in a straight line with acceleration $a = (6t - 8)\ \mathrm{m\ s^{-2}}$. Its initial velocity is $5\ \mathrm{m\ s^{-1}}$. Find the time T at which its velocity is next $5\ \mathrm{m\ s^{-1}}$, and its displacement from its starting point at time T. [5 marks]

Answer .....

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D1 A particle P moves on a straight line through O. At time t seconds its acceleration is $(8 - 2t)\ \text{m s}^{-2}$. At $t = 0$, P is at O with velocity $-12\ \text{m s}^{-1}$.

(a) Find an expression for the velocity of P at time t. [3 marks]

Answer .....

(b) Find the times at which P is at rest. [2 marks]

Answer .....

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(c) Show that P is back at O when $t = 6$. [3 marks]

(d) Find the total distance travelled by P in the first 7 seconds. [4 marks]

Answer .....

Total for D1: 12 marks

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D2

> Kim says: "$v = 2$ at $t = 0$ and $v = 10$ at $t = 2$, so the average velocity is $6\ \text{m s}^{-1}$ and the distance in the first 2 seconds is $6 \times 2 = 12\ \text{m}$."

A particle starts at O and moves in a straight line with acceleration $3t^2\ \text{m s}^{-2}$ at time t seconds. Its initial velocity is $2\ \text{m s}^{-1}$.

(a) Explain why Kim's method is not valid. [2 marks]

(b) Find the distance travelled in the first 2 seconds. [3 marks]

Answer .....

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(c) Find the time at which the velocity is $29\ \mathrm{m\ s^{-1}}$. [2 marks]

Answer .....

(d) Find the displacement from O at that time. [2 marks]

Answer .....

Total for D2: 9 marks

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D3 A particle moves on the x-axis. At time t seconds its acceleration is $(2t - 6)\ \mathrm{m\ s^{-2}}$. At $t = 0$ the particle is at O with velocity $10\ \mathrm{m\ s^{-1}}$ in the positive x-direction. Show that the particle never comes to rest, and find its displacement from O when its speed is least. [8 marks]

Answer .....

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D4 A car starts from rest at a point A and moves along a straight road. For $0 \leq t \leq 6$ its acceleration is $\left(2 - \frac{1}{3}t\right)\ \mathrm{m\,s^{-2}}$. At $t = 6$ the brakes are applied and the car then slows down with constant deceleration $4\ \mathrm{m\,s^{-2}}$ until it stops.

(a) Find the speed of the car when $t = 6$. [3 marks]

Answer .....

(b) Find the distance the car travels in the first 6 seconds. [3 marks]

Answer .....

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(c) Find the time at which the car stops and the total distance from A to where it stops. [3 marks]

Answer .....

(d) Explain why the constant acceleration formulae may be used for the second stage but not for the first. [2 marks]

Total for D4: 11 marks

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D5 Two particles P and Q move along the same straight line. At time $t = 0$ both are at the point O. P starts from rest at O, and its acceleration at time t seconds is $(16 - 6t)\ \mathrm{m\ s^{-2}}$. Q moves with constant velocity $16\ \mathrm{m\ s^{-1}}$ in the positive direction.

(a) Find expressions for the velocity of P and the displacement of P from O at time t. [3 marks]

Answer .....

(b) Find the times at which P and Q have the same velocity. [3 marks]

Answer .....

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(c) Show that, for $t > 0$, P and Q are level only when $t = 4$. Hence explain why P does not overtake Q. [4 marks]

Total for D5: 10 marks

D6 A particle P moves in a straight line. At time t seconds its acceleration is $(4 - 6t)\ \text{m s}^{-2}$. When $t = 1$, P has velocity $5\ \text{m s}^{-1}$. When $t = 2$, P is $10\ \text{m}$ from a fixed point O on the line, on the positive side.

(a) Find the velocity of P at time t. [3 marks]

Answer .....

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(b) Find the displacement of P from O at time t. [3 marks]

Answer .....

(c) Find the time at which P is at rest. [2 marks]

Answer .....

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(d) Find the time at which P is back at the point where it was when $t = 0$. [3 marks]

Answer .....

Total for D6: 11 marks

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E Extension

Stretch

No route is given. The hints are at the end of the sheet. Use one at a time.

E1 A particle starts from rest at O and moves in a straight line. At time t seconds its acceleration is $(2t - k)\ \mathrm{m\ s^{-2}}$, where k is a positive constant. The particle is back at O when $t = 6$. Find k, the greatest distance of the particle from O for $0 \leq t \leq 6$, and the total distance it travels in the first 8 seconds. [10 marks]

Answer .....

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Extension hints

Cover these with paper. Uncover one at a time.

E1

Hint 1 Integrate twice. The particle starts from rest at O, so both constants are zero. "Back at O" means $s = 0$.

Hint 2 The furthest point from O is where the particle turns round, where $v = 0$. Check the end $t = 6$ too.

Hint 3 For the distance in 8 seconds, split at the time it turns round and add the sizes of the two moves.

How did it go?

Colour one circle on each line: not yet, nearly, yes.

I can integrate to find velocity and displacement, using given conditions to find each constant.	○ ○ ○
I can find the distance travelled when a particle changes direction.	○ ○ ○
I can solve problems where the motion has two stages.	○ ○ ○
I can derive the constant acceleration formulae and explain when they cannot be used.	○ ○ ○

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